Technology Adoption, Innovation, and Inequality in a Global World*

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Abstract

I develop a tractable semi-endogenous multi-country growth model with a technology adoption margin. Innovation and adoption are skill-intensive activities, and a tradeoff arises whether skilled labor is used to push out the technological frontier or to adopt existing technology. I use the theory to revisit the effect of market integration on growth, especially among asymmetric countries with large differences in innovative capacity. Transitional dynamics and long-run effects implied by the model differ substantially from benchmark endogenous growth models and jointly explain stellar per capita growth in emerging markets and the disappointing growth performance of advanced economies after rising global market integration since the 1990s.

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1 Introduction

A robust implication of idea based growth models is that market integration boosts productivity growth. This is true even in the absence of cross-country knowledge spillovers, and when countries have unequal innovative capacity, since specialization in innovation vis-a-vis production accelerates technological change, see Grossman and Helpman (1991a). The surprisingly weak growth performance of advanced economies in the aftermath of market integration with emerging markets since the 1990s appears at odds with models of endogenous technological change. In this paper I develop a semi-endogenous growth model consistent with the growth performance of both advanced economies and emerging markets after a major globalization shock, which revisits the relationship between market integration and growth.

The novel feature of my model is to embed a technology adoption friction into an otherwise standard semi-endogenous growth model. The key assumption throughout is that both the invention of new ideas and the adoption of existing ones require the same scare factor, skilled labor. This creates a tradeoff between adopting technology vis-a-vis pushing out the technological frontier. This tension becomes particularly acute during an episode of market integration between advanced economies and emerging markets. In the integrated equilibrium, emerging markets adopt advanced economies' technology. To the extent that there is some patent protection, technology adoption abroad raises the returns to innovation and induces a reallocation of skilled labor from adoption toward innovation within advanced economies. Since aggregate growth depends on both the technological frontier and the degree of technology adoption, positive innovation effects can be entirely undone by weak technology adoption explaining uneven and sluggish growth in advanced economies as emerging markets join the world economy. Formalizing and exploring this mechanism by means of a tractable general equilibrium model is the main contribution of the paper.

My analysis proceeds in three steps. In the first step, I develop a closed economy version of the model. I build on Romer (1990)'s two-sector idea-based growth model where firms in the research sector invent new technology using skilled labor. Firms in the production sector combine idea-embodying capital goods with production labor to produce goods. My key departure is to introduce a technology adoption friction in the production sector whereby adopting new capital goods at the firm level is a costly and skill-intensive activity giving rise to an endogenous technology adoption gap.

The allocation of skilled labor across the two activities is generically inefficient. In addition to the standard externalities associated with innovation, technology adoption features a spillover as entering firms learn from incumbents' technology adoption choices.[1] Two countervailing forces give rise to a unique balanced growth path characterized by constants shares of skilled labor devoted to

[1] I will argue in detail that such a learning spillover is a necessary ingredient to generate a balanced growth path where productivity growth is partly driven by incumbent firms' technology adoption.

innovation and adoption, a constant technology adoption gap, and an ever-expanding technological frontier. First, the model features a complementarity between adoption and innovation on the market for ideas as they become profitable only after they are adopted. Higher adoption effort in the production sector thus pushes up the net present value of developing an idea. At the same time, by virtue of modeling both innovation and adoption as skill-intensive activities, a factor market rivalry arises as innovators and adopters compete for the same scarce resource, skilled labor.

In the second step, I generalize the model to a multi-country version to study economic growth in an integrated world. Building on Nelson and Phelps (1966), the technology adoption margin gives rise to a stationary cross-country income distribution. All countries grow at the same rate in the long-run and countries with greater research productivity, a larger population, or a relatively higher skilled labor share contribute more to the technological frontier. Even so, a country's productivity is unrelated to its size since small countries adopt technology invented elsewhere. There is no size advantage in adoption, and the skilled labor share is the key determinant of a country's position on the global income distribution.[2]Frictionless trade in undifferentiated final goods takes place whenever firms in a country adopt technology invented elsewhere to compensate the owner of technology.

In the third step, I quantify the effect of a market integration shock on growth.[3] I draw a sharp distinction between market integration amongst similar countries, like the formation of the European Common market in the second half of the 20th century, and asymmetric market integration between emerging markets and advanced economies, which I also refer to as "East" and "West". Symmetric integration has no adverse effects on adoption and delivers the standard gains from integration. The reason is that foreign adoption, which raises the incentive to innovate, and foreign innovation, which reduces it, exactly cancel.[4]

This contrasts with the East-West scenario, which I view as the distinct feature of the 1990s/2000s. I consider a thought experiment where each economy is on a balanced growth path, and an unanticipated liberalization moves each country from autarky to free trade in final goods and technology, which is meant to capture the fall of the Iron Curtain between advanced European economies and Eastern Europe, and China's ascension to the World Trade Organization in 2001. To discipline the quantitative exercise I combine cross-country data supplemented with micro data from Germany, which serves as a useful case study.[5]

The model generates a sizable increase in employment of skilled labor in the research sector and

[2]A model of this type suggest a negative link between the skill premium and GDP per capita. The link between the relative supply of skilled labor and the skill premium across countries is rather weak, see Caselli and Coleman (2006). I will address later how the model can be reconciled with this observation.

[3]As in Jones (1995) the long-run growth rate is a function of exogenous population growth, hence all growth effects induced by market integration are transitional.

[4]The result is reminiscent of Krugman (1980) and Eaton and Kortum (2001) where the equilibrium measure of firms or research effort is unrelated to trade costs

[5]The German economy is a major producer of technology, and underwent a sudden globalization shock after the fall of the Iron Curtain. The focus on Europe is due to data availability and a relatively clean shock, but the mechanism and implications carry over the relationship between the USA and China in the early 2000s.

patenting activity, consistent with the data. This drives up the skill premium by 25%, which leads production sector firms to cut down on skilled labor for adoption purposes, ultimately widening the technology adoption gap in the West by 20%. The calibration predicts a cumulative drop in real wages of production workers of 13%, relative to the counterfactual balanced growth path in autarky. While skilled labor gains in real terms, after aggregating up worker incomes within advanced economies, I arrive at a cumulative growth drag of about 9% for the aggregate wage bill. The impact of GDP per capita is around minus 5%, and the differences is accounted for by the role of asset accumulation and royalties earned abroad. High innovative activity, and an increase in the valuation of technology coincides with sluggish per capita growth as the economy transitions from one steady state to another. Even though productivity growth is tied to the evolution of the technological frontier in the long run, technology adoption dynamics are divorced from frontier growth along this transition path.

It is worth noting that in no way is a growth slowdown hard-wired into the model. The effect of asymmetric market integration on growth depends on whether the autarky allocation was characterized by an inefficiently low degree of technology adoption, a point emphasized in Perla, Tonetti, and Waugh (2021). Only if there is a misallocation of resources to begin with, which is then amplified in the open economy, will a temporary growth slowdown become possible. I show in detail how the results depend on the externalities in innovation and technology adoption, and the overall gains from trade. Loosely speaking, if ideas are harder to find (Bloom et al., 2020), and there are learning externalities in adoption, an inefficient autarky allocation becomes a distinct possibility.[6] This inefficiency is amplified when integrating with a foreign economy that adopts technology aggressively but contributes little frontier technology themselves. A corollary of this is that full convergence of the East in terms of skill endowment and innovative capacity would restore the powerful pro-growth effects of market integration, and simultaneously reduce the skill bias in the West.

Lastly, I provide suggestive evidence in favor of the key mechanism, and address related issues such as skill biased technological change, automation, and alternative explanations for the growth slowdown. Models of skill-biased technological change and automation are hard to reconcile with the growth slowdown, and the endogenous growth literature has almost exclusively focused on frontier innovation. Little attention has been paid to the role of technology adoption in a globalized world,[7] which offers a simple explanation for a broad set of facts.

Relationship to the literature. I build on the literature on endogenous growth (Romer, 1990; Grossman and Helpman, 1991b; Aghion and Howitt, 1990), and incorporate a technology adoption margin into Jones (1995). A large literature uses variants of these models to study the aggregate productivity

[6] I derive a sufficient statistic for the long-run impact of integration on wages similar to Arkolakis, Costinot, and Rodríguez-Clare (2012). This statistic is extremely sensitive to how the dynamic knowledge externality is parameterized and how important technology adoption is.

[7] Jovanovic (1997) argues that costs of technology adoption as a share of GDP exceed costs associated with innovation by an order of magnitude.

slowdown,[8] focusing on falling population growth and declining business dynamics as drivers.[9] Relative to this literature, I focus on the distinction between innovation and technology adoption in the context of an asymmetric globalization shock.[10]

Second, a large literature studies the role of technology adoption for cross-country income differences, see Parente and Prescott (1994), Lucas (2009a), Comin and Hobijn (2010a), or Comin and Mestieri (2014). Barro and Martin (1997) and Acemoglu, Aghion, and Zilibotti (2006) study developing economies' choice between adoption and innovation.

A small number of papers has modeled innovation and adoption jointly. Konig et al. (2021), building on König, Lorenz, and Zilibotti (2016), as well as Benhabib, Perla, and Tonetti (2021), Hopenhayn and Shi (2020), and Sampson (2023) develop heterogeneous firm models where high productivity firms innovate, while laggard firms imitate high productivity firms. Comin and Gertler (2006) and Anzoategui et al. (2019) model innovation and adoption jointly over the business cycle. The complementarity between innovation and adoption on the market for ideas is related to but distinct from Comin and Hobijn (2007), which focuses on initial implementation of new technologies, as well as Benhabib, Perla, and Tonetti (2021)'s case of licensing agreements, following Hopenhayn and Shi (2020). Relative to these works, I draw out an innovation-adoption tradeoff embedded in an otherwise standard semi-endogenous growth model with realistic scale effects.

Third, I relate to the large literature on trade and growth. Rivera-Batiz and Romer (1991) highlight the strong pro-growth effect of market integration. Eaton and Kortum (1999) develop a model of global growth with exogenous diffusion of ideas. Sampson (2016) and Perla, Tonetti, and Waugh (2021) highlight the role of learning and firm heterogeneity in Melitzian settings. Buera and Oberfield (2020) model the global diffusion of ideas, while I focus on forward-looking adoption choices. Relative to the large quantitatively focused trade literature,[11] I keep the model simple to draw out the special role of technology adoption in the presence of an asymmetric market integration shock.

The rest of the paper proceeds as follows: Section 2 presents a model of innovation and adoption. Section 3 introduces the open economy version. Section 4 quantifies the model and studies transition

[8] The productivity slowdown is a robust feature of the data, although its onset differs somewhat across countries. Fernald (2015) and Cette, Fernald, and Mojon (2016) point out that this slowdown started before the financial crisis.

[9] See Peters and Walsh (2021), Jones (2020), Engbom et al. (2019), and Hopenhayn, Neira, and Singhania (2018) on works that highlight the role of population growth on productivity and business dynamics. An alternative explanation for the productivity slowdown focuses on firm dynamics and biased technology shocks, see De Ridder (2019), Olmstead-Rumsey (2019), Rempel (2021), Akcigit and Ates (2019), and Aghion et al. (2019), all of which are building on Klette and Kortum (2004). Incorporating an endogenous technology adoption margin along the lines proposed here should be complementary to the overall agenda in this subfield.

[10] The theory shares some predictions with models of directed technological change (Acemoglu, 2002; Acemoglu, 2003; Acemoglu, Gancia, and Zilibotti, 2015), models of appropriate technology (Atkinson and Stiglitz, 1969; Basu and Weil, 1998; Acemoglu and Zilibotti, 2001; Caselli and Coleman, 2006), and models of technological revolutions (Greenwood and Yorukoglu, 1997; Caselli, 1999).

[11] The steady state of the model in principal allows for additional sources of differentiation, richer trade costs, and technology frictions as in Arkolakis et al. (2018). What complicates the analysis are forward-looking innovation and adoption choices off the steady state. For more quantitatively focused work on the link between trade and growth see Lind and Ramondo (2022), Cai, Li, and Santacreu (2022), and Somale (2021) building on Eaton and Kortum (1999).

dynamics. Lastly, I discuss empirical evidence consistent with the key mechanism and conclude.

2 Closed Economy

2.1 Environment

I outline the economic environment next, which is cast in continuous time. All derivations and proofs are deferred to the appendix.

Households. A representative household supplies their labor inelastically, which leads to an economy wide endowment of L units of production labor and H units of high skilled labor. The household's labor endowment grows at rate $g_H = g_L \geq 0$. Factors earn income at wage rates $w_{L,t}$ and $w_{H,t}$, and the skill premium is defined as $s_t := \frac{w_{H,t}}{w_{L,t}}$. The household solves a consumption-saving problem

$$\begin{aligned} U &= \max_{\{c_t, B_t\}_{t\geq 0}} \int_0^\infty e^{-(\rho - g_L)t} \log c_t \, dt \\ \text{s.t.} & \\ \dot{B}_t &= r_t B_t + w_{H,t} H_t + w_{L,t} L_t - C_t, \end{aligned} \tag{1}$$

with the usual transversality condition in place. Total assets in the economy are denoted by B. Changes in total assets $\dot{B}$ represent net savings while r is the return on assets. Per capita consumption growth follows from the solution to (1) and reads $\frac{\dot{c}}{c} = r_t - \rho$.

Production Sector. A competitive final goods producer combines differentiated intermediate goods using a Benassy (1996)-CES aggregator

$$Y_t = M_t^{-\delta_Y} \left(\int y_{i,t}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where M is the measure of intermediate goods and $\delta_Y = \frac{1}{\sigma-1}$.[12] Final output, which serves as the numeraire, can be consumed or turned into an investment good

$$Y_t = C_t + I_t, \tag{3}$$

where C_t is aggregate consumption. Physical capital depreciates at rate δ_k

$$\dot{K} = I_t - \delta_k K_t. \tag{4}$$

Monopolistically competitive infinitesimal firms $i \in \Omega_{M_t}$ produce differentiated intermediate

[12] This assumption ensures that there are no scale effects in the production sector. An alternative setup building on Hopenhayn (1992) is presented in the appendix where no such assumption is needed to maintain a production sector that is constant-returns-to-scale. The payoff here is that the constant-returns-to-scale production function on the firm level makes the exposition easier.

goods using capital goods $x_{ij,t} \in \Omega_{A_{i,t}}$ and production labor $l_{i,t}$ according to

$$y_{i,t} = \left(\int_{j \in \Omega_{A_{i,t}}} \left(\frac{x_{ij,t}}{\alpha} \right)^{\alpha} dj \right) \left(\frac{l_{i,t}}{1-\alpha} \right)^{1-\alpha}. \tag{5}$$

Production labor is rented at rate $w_{L,t}$, and differentiated capital goods are rented at rate $p_{xj,t} = p_{x,t}$ since capital goods are assumed to be symmetric. This leads to a simple static cost minimization problem where the firm takes as given the set $\Omega_{A_{i,t}}$ and factor prices to minimize production cost given (5). A solution to this problem requires the firm to spread its capital expenditure evenly across different capital goods

$$p_{x,t} x_{ij,t} = \frac{p_{x,t} X_{i,t}}{A_{i,t}}, \tag{6}$$

where $A_{i,t} \in R^{+}$ is the measure of set $\Omega_{A_{i,t}}$ and $X_{i,t} = \int x_{ij,t} dj$. Inserting this back into (5) delivers the marginal cost of production $\mathrm{mc}_{i,t} = (p_{x,t})^{\alpha} \left(\frac{w_{L,t}}{A_{i,t}} \right)^{(1-\alpha)}$ where $A_{i,t}$ can be interpreted as firm productivity. Moreover, given iso-elastic demand for intermediate goods from (2) and monopolistic competition,[13] a constant markup over marginal cost applies $p_{i,t} = \frac{\sigma}{\sigma-1} \mathrm{mc}_{i,t}$. Firm operating profits, which are the difference between sales relative to production cost excluding overhead adoption costs, follow

$$\pi^{o}_{i,t} = \frac{Y_{i,t}}{\sigma} \left(\frac{\sigma}{\sigma-1} p_{x,t}^{\alpha} w_{L,t}^{1-\alpha} \right)^{1-\sigma} A_{i,t}^{(1-\alpha)(\sigma-1)}. \tag{7}$$

Technology adoption in this model is equivalent to making new capital goods available to the firm in the production sector, i.e., expanding the set $\Omega_{A_{i,t}}$. Relative to the set $\Omega_{A_{\mathrm{F},t}}$, the superset $\Omega_{A_{\mathrm{F},t}}$ contains all existing capital goods including recent inventions not adopted yet, and the associated measure $A_{\mathrm{F},t}$ can be interpreted as the technological frontier. Following Nelson and Phelps (1966), I make two crucial assumptions. First, technology adoption requires skilled labor. Second, technology adoption features an "advantage of backwardness". Equation (8) formalizes these concepts

$$\dot{A}_{i,t} \quad = \quad \nu A_{\mathrm{F},t}^{1-\theta} A_{i,t}^{\theta} h_{i,t}^{\beta} - \delta_{\mathrm{I}} A_{i,t}, \tag{8}$$

where $h_{i,t}$ is the amount of skilled labor hired by firm i for adoption purposes, $1-\theta \in (0,\infty)$ governs the advantage of backwardness, and the parameter $\beta \in (0,1)$ induces diminishing returns to technology adoption at a point in time. A random Poisson death shock $\delta_{\mathrm{I}} \geq 0$ hits capital goods, and $\nu > 0$ is

[13] Since technology adoption is costly and firms produce according to a constant-returns-to-scale technology, I have to deviate from the benchmark competitive production sector. This is, of course, the same argument put forth in Schumpeter (1942) and Romer (1990) as to why constant-returns-to-scale competitive economies are not consistent with theories of endogenous growth.

a constant assumed to be sufficiently small to avoid a corner solution at $A_{i,t} = A_{\mathrm{F},t}$.[14]

I close the production sector by assuming free entry after paying a fixed cost f_{E} in terms of production labor

$$\int V_t(A)\, dF_t(A|\mathrm{E}) \leq f_{\mathrm{E}} w_{L,t}, \tag{9}$$

where $V_t(A)$ is the value of a production sector firm of productivity A. The expected present discounted value of entry must not exceed the cost of entry, where $dF_t(A|\mathrm{E})$ is the conditional probability distribution over productivity levels of entering firm. I assume that the distribution $F_t(A|\mathrm{E})$ is tied to the incumbent distribution $F_t(A)$ modeled as a linear function of average incumbent productivity

$$A_{\mathrm{E}} = \lambda_{\mathrm{E}} \int A dF_t(A)$$

where $\lambda_{\mathrm{E}} \in (0, 1]$.[15] Because incumbents are not compensated for this knowledge spillover, their equilibrium adoption effort may be too low, which is an issue I will return to. This contrasts with the standard heterogenous firm model of Hopenhayn (1992) or Melitz (2003) where the distribution $dF_t(A|\mathrm{E})$ is exogenous and fixed. Note that in any growth model where private firms' technology adoption is a key ingredient of long-run growth, a learning spillover from incumbents to entrants is a necessary feature of the environment whenever there is positive firm entry. To see why, imagine that incumbents improve their productivity at a constant rate by adopting new technology, but entrants enter at some fixed productivity level. In that scenario the firm size distribution would be ever diverging. To obtain a stable firm size distribution, a spillover from entrants to incumbents is needed, which relates to the recent idea flow literature, especially Luttmer (2007).[16]

Assumption 1. *Let* $\lambda_E = 1$.

After imposing this strong knowledge spillover similar to Sampson (2016) or Perla and Tonetti (2014), the setup collapses conveniently to a homogenous firm model allowing me to drop all i subscripts.[17] Since entrants are, on average, as productive as incumbents, and technology adoption in (8) has no exogenous firm-specific parameters, the only fixed point is one where firms make identical choices.

Research Sector. Following Romer (1990), innovators produce a flow $\frac{1}{f_{\mathrm{R},t}} A^{\phi}_{\mathrm{F},t}$ of new ideas with one

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[14] An alternative is to use the original Nelson-Phelps specification $\dot{A}_{i,t} = (A_{\mathrm{F},t} - A_{i,t})\, g(h_{i,t})$ for some monotone function g, which does not change any qualitative insights of the model but ensures that no matter how much skilled labor is used, the firm never chooses a corner solution. My specification has an additional degree of freedom in θ which allows me to match the speed of convergence across countries. See Lucas (1993), Parente and Prescott (1994), and Sampson (2023) for similar formulations of technology adoption.

[15] Alternatively, one could also use the shifted distribution $F_t(A|\mathrm{E}) = F_t\left(\frac{A}{\lambda_{\mathrm{E}}}\right)$ where for λ_{E} and some lower bound $\underline{A}_{\mathrm{E},t}$ a non-degenerate distribution emerges.

[16] See also Jovanovic and Rob (1989) and Lucas (2009b) for seminal works.

[17] Even for the case of $\lambda_{\mathrm{E}} = 1$, however, the model is flexible enough to entertain the limiting case of efficient technology adoption. Since the extent of the spillover is directly related to the amount of entry by new firms, shutting down entry by changing exogenous parameters like death shocks or population growth will rid the model of the adoption externality.

unit of skilled labor, where f_{R} is a fixed entry cost, A_{F}^{ϕ} represents a knowledge spillover with $\phi \in (-\infty, 1)$ governing the strength of the spillover as in Jones (1995). Moreover, ideas die at Poisson rate δ_{I}, which leads to the following law of motion of the technological frontier

$$\dot{A}_{\mathrm{F},t} = \frac{1}{f_{\mathrm{R},t}} A_{\mathrm{F},t}^{\phi} H_{\mathrm{F},t} - \delta_{\mathrm{I}} A_{\mathrm{F},t}, \tag{10}$$

where H_{F} denotes the total amount of skilled labor devoted to innovation. The technological frontier A_{F} comprises all existing ideas, adopted or not, which are symmetric. The fixed cost $f_{\mathrm{R},t} = \frac{\left(\frac{H_{\mathrm{F},t}}{L_t}\right)^{1-\lambda}}{\gamma}$ depends on exogenous research productivity γ, and a congestion externality $\left(\frac{H_{\mathrm{F},t}}{L_t}\right)^{1-\lambda}$ parameterized by $\lambda \in (0, 1]$. For $\lambda < 1$, innovation becomes harder as the share of labor devoted to innovation increases.[18] After invention, innovators hold on to an infinitely lived patent,[19] and rent out differentiated capital goods to intermediate goods producers. I follow Romer (1990) and assume that manufacturers of differentiated varieties rent physical capital from households and combine it with their unique patent to finally rent out differentiated capital goods to production sector firms at price $p_{x,t}$.

To pin down the present discounted value of an innovation, forward-looking innovators take into account that their ideas become profitable only after they are adopted, which takes time and is the main difference to Romer (1990). This feature will generate a feedback between innovation and adoption. For simplicity, I assume that ideas are adopted according to when they were invented, and older ideas are adopted first.[20] This imposes a natural ordering of idea adoption, which leads to a tractable expression of the waiting time $\tau_t \in R^+$ for an idea to be adopted. The waiting time is key as it will impact the present discounted value of an innovation: if it took forever for an idea to be adopted, the value of innovation would be zero.

2.2 Solving the model

I assume monopolistic competition in both the production sector and the research sector. I first solve the dynamic problem of firms in the production sector, taking the evolution of the frontier as given. I then turn attention to the research sector and solve for the evolution of the technological frontier taking technology adoption choices as given. An equilibrium is defined as a fixed point where forward-looking profit maximizing choices in each sector are consistent, and markets clear. Additional details and derivations are deferred to the appendix.

Production Sector. Firms in the production sector face a dynamic tradeoff between the cost of skilled

[18] The congestion force is slightly different from the one in Jones (1995) where $f_{\mathrm{R},t} = \frac{(H_{\mathrm{F},t})^{1-\lambda}}{\gamma}$ represents the possibility of useless duplication, i.e., two researchers coming up with the same idea at the same time. The sort of congestion I have in mind here is better thought of as a reduced form way of taking account of heterogeneity in research talent as in Phelps (1966). The specification is chosen with an eye toward the open economy, and delivers a well-behaved open economy equilibrium.

[19] Although note that there are Poisson death shocks so most patents vanish eventually.

[20] All ideas, already adopted or waiting to be adopted, are subject to the death shocks, which retains tractability.

labor today, and the future benefit of adopting technology, which can be formalized using an HJB equation. When using the HJB equation, it is convenient to to use a normalized level of technology relative to the technological frontier $z_t := \frac{A_t}{A_{\mathrm{F},t}}$, and normalize the firm value function V by the wage $w_{L,t}$, i.e., $v_t := \frac{V_t(z_t)}{w_{L,t}}$. This equivalent but potentially stationary dynamic program reads

$$\begin{aligned} v_t\left(r_t - g_{w_L,t} + \delta_{\mathrm{X}}\right) &= \max_{h_t} \frac{\pi_t^0(z)}{w_{L,t}} - s_t h_t + \dot{z}_t \partial_z v + \dot{v} \\ \text{s.t.} & \\ \dot{z}_t &= \nu z_t^\theta h_t^\beta - \left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right) z_t, \end{aligned} \tag{11}$$

where g_{w_L}, $\frac{\pi_t^0(z)}{w_L}$, $\partial_z v$, $\dot{v}$, g_{F} represent production wage growth, normalized profits, partial derivative of the value function with respect to z, time derivative, and growth rate of frontier technology. I now drop time subscripts for readability. An interior solution of (11) satisfies the first order condition

$$\left[\frac{\beta\zeta z^\theta \partial_z v}{s}\right]^{\frac{1}{1-\beta}} = h, \tag{12}$$

where the skill premium appears as the key relative price associated with the cost of technology adoption while the term $\beta\zeta z_i^\theta \partial_z v$ captures the marginal benefit. Totally differentiating (12) in combination with an envelope condition delivers a differential equation governing technology adoption

$$\frac{\dot{h}}{h} = \frac{1}{1-\beta}\left\{r_t - g_{w_L} + \delta_{\mathrm{X}} + (1-\theta)\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) - \frac{\dot{s}}{s} - \frac{\beta h^{\beta-1}\nu z^\theta}{s}\left[\frac{\pi^o}{w_L}\frac{(1-\alpha)(\sigma-1)}{z}\right]\right\}, \tag{13}$$

which summarizes the tradeoff for a firm in the production sector: the left hand side represents an effective discounting term consisting of interest rate, wage growth, and death shocks, and a less standard term accounting for the advantage of backwardness $(1-\theta)\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right)$. Intuitively, if the advantage of backwardness is strong, delaying investment in technology adoption is beneficial inducing a higher effective discount factor. Similarly, if there is an anticipated increase in the relative price of skilled labor $\frac{\dot{s}}{s}$, investing in technology adoption is especially advantageous today relative to tomorrow, pushing down the effective discount factor. The term $\frac{\beta h_i^{\beta-1}\nu z_i^\theta}{s}\left[\frac{\pi_i^o}{w}\frac{(1-\alpha)(\sigma-1)}{z_i}\right]$ captures the net present discounted value of an extra unit of skilled labor, which depends on the elasticity of firm profits with respect to relative productivity, $\frac{\partial \pi^o}{\partial z} = \pi^o\frac{(1-\alpha)(\sigma-1)}{z}$, as well as the marginal effect of an increase in skilled labor on adoption, $\beta h_i^{\beta-1}\nu z_i^\theta$.

Given assumption 1, the value function admits a closed form solution on and off the balanced growth path whenever the entry condition is binding

$$v = \frac{\frac{\pi^o}{w_L} - sh}{r - g_{w_L} + \delta_{\mathrm{X}}}. \tag{14}$$

Lastly, imposing free entry $v \leq f_{\mathrm{E}}$ pins down entry into the production sector.

Research Sector. As in Romer (1990), the owner of intellectual property applies a constant markup to the cost of supplying a unit of the differentiated capital good,[21] which equals the rental rate plus capital depreciation $r+\delta_k$, leading to $p_x = \frac{1}{\alpha}(r+\delta_k)$. For adopted technology, this leads to a royalty π_{I}, which, after some simplification and applying the pricing rule $p_x = \frac{r+\delta_k}{\alpha}$, read $\pi_{\mathrm{I}} = \alpha \frac{L_{\mathrm{P}} w_L}{A}$. The present discounted value of an innovation thus follows as

$$V_{\mathrm{I}} \quad = \quad \int_{t+\tau_t}^{\infty} \exp\left(-\int_t^u (r_v+\delta_{\mathrm{I}})\,dv\right)\pi_{\mathrm{I}}(u)\,du, \tag{15}$$

where the discount factor, $r+\delta_{\mathrm{I}}$, runs from t onward although profits accrue only form $t+\tau$ with τ referring to the waiting time until an idea is adopted.

The equilibrium amount of skilled labor devoted to innovation H_{F} is pinned down by a free entry condition

$$V_{\mathrm{I}} \quad \leq \quad \frac{f_{\mathrm{R}} w_H}{A_{\mathrm{F}}^{\phi}} \quad . \tag{16}$$

Whenever the free entry condition in (16) is binding, a closed-form solution for the value of an innovation at time of entry obtains. Differentiating both (16) and (15) with respect to time and combining them yields

$$V_{\mathrm{I}} = \frac{\exp\left(-\int_t^{t+\tau}(r_v+\delta_{\mathrm{I}})\,dv\right)\pi_{\mathrm{I},t+\tau}\cdot[1+\dot{\tau}_t]}{r-g_{w_L}+\delta_{\mathrm{I}}-g_s-(1-\lambda)(g_{H_{\mathrm{F}}}-g_L)+\phi g_{\mathrm{F}}}, \tag{17}$$

where a standard discount factor $\frac{1}{r+\delta_{\mathrm{I}}-g_w-g_s-(1-\lambda)\left(g_{H_{\mathrm{F}}}-g_L\right)+\phi g_{\mathrm{F}}}$ accounts for firm death, time discounting, and appreciation of the value of an innovation implied by free entry into the research sector. Non-standard is the term $\exp\left(-\int_t^{t+\tau}(r_v+\delta_{\mathrm{I}})\,dv\right)\pi_{\mathrm{I},t+\tau}\cdot[1+\dot{\tau}_t]$, which accounts for the fact that profits $\pi_{\mathrm{I},t+\tau}$ arrive with a delay, and the delay itself could change over time $\dot{\tau}_t$. An increase in the waiting time necessitates higher firm profits to respect the break-even condition implied by free entry.

It is clear from the previous results that innovation depends on adoption through the endogenous waiting time τ, which an innovator needs to forecast to decide whether they should enter the research sector. The assumption that ideas are adopted according to when they were invented[22] in-

[21] In this model the capital share and the markup are tied together. One could easily change this by modeling the production function of intermediate goods firms using a double-nest with two different elasticities, see Jones and Williams (2000). This changes the increasing returns on the firm level, and would require different parameters values for λ and ϕ to match the same long-run TFP series in the data.

[22] Simply put, innovators wait in line till they are up. And they are up when all innovators that invented before them are either adopted or disappeared due to the death shock. Implicit in this model is that all firms in the production sector adopt technology in the same order. A version with stochastic adoption would break this result, which I sketch out in the appendix, and similarly, in the heterogeneous firm version different technology would arrive in different firms at the same point in time. I emphasize that whether the adoption is deterministic or stochastic is not central for any of the results in the paper. Since markets are complete, idiosyncratic risk washes out in the aggregate. What matters for the key mechanism in the paper is that the average waiting time is a function of adoption effort in the production sector.

duces a simple law of motion for the waiting time that makes this forecasting problem tractable. Formally, define the measure of ideas which stands between the adoption of some cohort t's innovation $W(t,t) := A_{\mathrm{F}} - A$, where the first argument refers to the time when cohort t paid the fixed cost to innovate. The calendar time of adoption for inventor cohort t is, by definition, $t + \tau_t$. First, note that while new ideas may be invented, they will only be adopted after cohort t and are thus irrelevant for cohort t's waiting time τ_t. What matters for cohort t is how quickly the measure W melts away over time, which depends on two factors: first, ideas die at rate δ_{I}, so a flow $W\delta_{\mathrm{I}}dt$ is disappearing every instant. Second, a flow $A_t(g_A + \delta_{\mathrm{I}})\,dt$ is adopted every instant with $g_A := \frac{\dot{A}}{A}$ being the net growth rate of ideas on the firm level.[23] The reduction in W over time thus obeys the differential equation

$$\dot{W} \quad = \quad -\delta_{\mathrm{I}} W - A\left(g_A + \delta_{\mathrm{I}}\right). \tag{18}$$

By definition, after waiting for τ years the waiting time is zero, i.e., $W(t, t+\tau_t) = 0$, so τ_t is implicitly defined by an initial condition $W(t,t) = A_{\mathrm{F},t} - A_t$, a terminal condition $W(t, t+\tau_t) = 0$, and a trajectory of A that depends on technology adoption. The setup leads to a simple solution for τ.

Proposition 1. *The endogenous waiting time τ_t, on and off the balanced growth path, reads*

$$\tau_t = -\frac{\log z_t}{\frac{\int_t^{t+\tau_t} g_A(x)dx}{\tau_t} + \delta_I}. \tag{19}$$

To study a balanced growth path later on, I introduce the normalized variables $v_{\mathrm{I}} := \frac{V_{\mathrm{I}}}{A_{\mathrm{F}}^{\phi} w_L}$, $a_{\mathrm{F}} := \frac{A_{\mathrm{F}}^{1-\phi}}{L}$, $\frac{L_{\mathrm{P}}}{L} := l_{\mathrm{P}}$, $h_{\mathrm{F}} := \frac{H_{\mathrm{F}}}{L}$. The normalized value of an innovation simplifies to

$$v_{\mathrm{I}} = \frac{\exp\left(-\int_t^{t+\tau}\left(r_v + \delta_{\mathrm{I}} + g_A - g_w - g_L\right)dv\right)}{r_t - g_w - g_s + \delta_I - (1-\lambda)\left(g_{H_{\mathrm{F}}} - g_L\right) + \phi g_{\mathrm{F}}} \frac{\alpha l_{\mathrm{P},t+\tau}}{z a_{\mathrm{F}}} \cdot \left[1 + \dot{\tau}_t\right] \tag{20}$$

and $v_{\mathrm{I}} = \frac{s h_{\mathrm{F}}^{1-\lambda}}{\gamma}$ whenever there is positive entry. The term $\frac{\alpha l_{\mathrm{P},t+\tau}}{z a_{\mathrm{F}}}$ captures a market size effect where a larger production sector $l_{\mathrm{P},t+\tau}$ raises profits, while more competition by other innovators ($z a_{\mathrm{F}}$) lowers them.

Normalized laws of motion for firm creation in the research and production sector read

$$\dot{a}_{\mathrm{F}} = (1-\phi)\left\{\gamma h_{\mathrm{F}}^{\lambda} - a_{\mathrm{F}}\left[\delta_{\mathrm{I}} + \frac{1}{1-\phi} g_L\right]\right\}, \tag{21}$$

$$\dot{m} = \frac{l_{\mathrm{E}}}{f_{\mathrm{E}}} - \left(g_L + \delta_{\mathrm{X}}\right) m, \tag{22}$$

[23] Note that in order to achieve net variety growth g_A the intermediate goods firm needs to adopt $A_t(g_A + \delta_{\mathrm{I}})\,dt$ varieties to make up for the loss of ideas due to the random death shock. A production firm will never drop ideas on purpose so a negative growth rate is bounded by $-\delta_{\mathrm{I}}$ when there is zero adoption.

with $m := \frac{M}{L}$, and $l_{\mathrm{E}} := \frac{L_{\mathrm{E}}}{L}$.
Market clearing. Normalized market clearing conditions are defined as

$$\begin{aligned} K &= X \\ Y &= C + I \\ 1 &= l_{\mathrm{E}} + l_{\mathrm{P}} \\ h_{\text{tot}} &= h_{\mathrm{F}} + h_{\mathrm{D}}, \end{aligned}$$

where $h_{\mathrm{D}} := \frac{H_{\mathrm{D}}}{L}$ and $h_{\text{tot}} := \frac{H}{L}$. I next study the balanced growth path of the decentralized solution in autarky.

2.3 Balanced Growth Path

Equilibrium Definition, Existence, and Uniqueness. The concept of a balanced growth path is very similar to Jones (1995). The main difference is that equilibrium in the production sector gives rise to a constant technology adoption gap, defined as $\Gamma := -\log z$.

Proposition 2. *Along a balanced growth path wages, per capita consumption, productivity, and the technological frontier grow at rate $g_A = g_{w_L} = g_{w_H} = g_F = \frac{1}{1-\phi} g_L$, while population grows at exogenous rate $g_L = g_H \geq 0$.* The endogenous variables *$z, s, a_F, m, l_E, l_P, h, h_D, h_F$ are constant and the interest rate equals $r = \rho + g_F$.*

With the exception of pathological corner solutions,[24] equilibrium is unique, if it exists.

Assumption 2. *Existence requires the following inequality to hold*

$$\rho + \delta_X + (1-\theta)(g_F + \delta_I) > \beta (g_F + \delta_I)(1-\alpha)(\sigma - 1).$$

This assumption ensures that the benefit of adoption is sufficiently small so that firms' net profits $\frac{\pi^o}{w_L} - sh$ are positive. If, instead, varieties were too substitutable ($\sigma \to \infty$), firms will want to upgrade their technology to capture the entire market. Spending on adoption would be so large that no profits were left to cover the cost of entry, and the equilibrium would unravel.

In a stationary equilibrium I have $\dot{z} = 0$ which implies $z = \left(\frac{\nu h^{\beta}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}\right)^{\frac{1}{1-\theta}}$. Together with (13) and $\dot{h} = 0$, the equilibrium demand of skilled labor of a firm in the production sector becomes a simple function of the skill premium and operating profits

$$h = \frac{1}{s} \frac{\beta (1-\alpha)(\sigma-1)(g_{\mathrm{F}} + \delta_{\mathrm{I}})}{\rho + \delta_{\mathrm{X}} + (1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})} \frac{\pi^o}{w_L}. \tag{23}$$

[24] Zero innovation and zero adoption effort would constitute such a case.

Using the free entry condition, one can show that the flow profits are a function of exogenous parameters, and I can simply write $h = \frac{1}{s}\Lambda_h$ where Λ_h picks up constant parameters. [25] Note that the number of skilled workers per firm in the production sector is constant and rising demand for skilled labor devoted to technology adoption stems from the extensive margin in the case of a growing population.

The equilibrium adoption gap is pinned down by h using $\dot{z} = 0$

$$z = \left(\frac{\nu \left(\frac{1}{s}\Lambda_h\right)^{\beta}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} \right)^{\frac{1}{1-\theta}},$$

which establishes an immediate link between the skill premium and technology adoption, which is the key result in the paper on which all other implications hinge.

Proposition 3. *An increase in the skill premium, ceteris paribus, leads to an increase in the technology adoption gap*

$$-\frac{\partial \log z}{\partial \log s} = \frac{\beta}{1-\theta}.$$

The endogenous response of the equilibrium adoption gap to a change in the skill premium seems both obvious and intuitive, yet has received little attention in the literature.[26] Note that the solution $h = \frac{1}{s}\Lambda_h$, together with free entry,[27] pins down the normalized measure of firms $m = \frac{1}{f_{\mathrm{E}}((\rho+\delta_{\mathrm{X}})(1-\alpha)(\sigma-1)\Lambda_\pi + \delta_{\mathrm{X}} + g_L)}$ where $\Lambda_\pi = \left(1 - \frac{\beta(1-\alpha)(\sigma-1)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\rho+\delta_{\mathrm{X}}+(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}\right)^{-1}$ is a constant. Using this, I can derive aggregate normalized demand for skilled labor devoted to adoption follows, $h_{\mathrm{D}} := \frac{H_{\mathrm{D}}}{L} = m \cdot h$, which is downward-sloping in the skill premium.[28]

Having understood how equilibrium adoption is pinned down along the balanced growth path, I zoom in on the complementarity between innovation and adoption on the market for ideas. The two

[25] Combining (23) with the free entry condition, $f_{\mathrm{E}} = \frac{\frac{\pi^o}{w_L} - sh}{\rho + \delta_{\mathrm{X}}}$, and using $r = \rho + g_w$, delivers the equilibrium demand for skilled labor on the firm level as a function of exogenous parameters and the skill premium $h = \frac{1}{s}\Lambda_h$ where $\Lambda_h = \frac{\beta(1-\alpha)(\sigma-1)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\rho+\delta_{\mathrm{X}}+(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})} f_{\mathrm{E}}(\rho+\delta_{\mathrm{X}})\Lambda_\pi$ with $\Lambda_\pi = \left(1 - \frac{\beta(1-\alpha)(\sigma-1)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\rho+\delta_{\mathrm{X}}+(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}\right)^{-1}$ being the factor by which the operating profits need to exceed the flow cost of entry to make up for the cost of technology adoption.

[26] Cummins and Violante (2002) is an important exception, which documents a widening gap between frontier technology and average technology level. I provide a micro-founded model of technology adoption where the skill premium pins down the technology adoption gap, consistent with the findings in their paper.

[27] Note $Y\frac{\sigma-1}{\sigma}(1-\alpha) = w_L L_{\mathrm{P}}$ due to Cobb-Douglas production, and $\pi^o = \frac{Y}{M}\frac{1}{\sigma}$, implies $\frac{\pi^o}{w_L} = \frac{l_{\mathrm{P}}}{m}\frac{1}{(1-\alpha)(\sigma-1)}$. Using this together with free entry $f_{\mathrm{E}}(\rho+\delta_{\mathrm{X}}) = \frac{\pi^o}{w_L} - sh$, the law of motion of firm creation $\dot{m} = \frac{l_{\mathrm{E}}}{m f_{\mathrm{E}}} - (\delta_{\mathrm{X}} + g_L) = 0$ in the steady state, and $l_{\mathrm{P}} = 1 - l_{\mathrm{E}}$, delivers the result.

[28] The size of the fixed cost of entering the production sector impacts technology adoption since h_i is proportional to firm profits. However, h_{D}, i.e., aggregate normalized demand for skilled labor in the production sector is unrelated to f_{E}. One could break the link between z and f_{E} by assuming that the parameter ν is proportional to the fixed cost $\nu = \nu_0 \cdot f_{\mathrm{E}}^{-\beta}$. The solution to the share of operating profits devoted to skilled labor remains unchanged, but the fixed cost will not show up in the adoption gap anymore. A richer model where technology adoption costs are proportional to the size of the firm, measured in workers or revenue, may be desirable. In such a model, the firm's static optimality conditions turn into dynamic ones as hiring more workers would reduce the effectiveness of technology adoption, complicating the setup substantially.

are tied together through the endogenous waiting time τ, which simply equals $-\frac{\log z}{g_A+\delta_{\mathrm{I}}}$ along a balanced growth path following from proposition 1. Substituting out τ when computing the normalized present discounted value of an innovation from (20) leads to

$$v_{\mathrm{I}} = \frac{1}{\rho + \phi g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}} z^{\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}} \tag{24}$$

where I use an effective discount factor $\tilde{\rho} = \rho - g_L$. The value of an innovation in (24) makes explicit the dependence on technology adoption. When there is no adoption ($z \to 0$), the value of an innovation is zero. As ideas are adopted more quickly, the value of innovation increases, and when $z \to 1$ the expression effectively nests the present discounted value in Romer (1990). In a free entry equilibrium, and holding everything else fixed, increasing technology adoption leads to rising innovation. To see this, combine equation (24) with the free entry condition, and the resource constraint for $\dot{a}_F$ to obtain equilibrium demand for skilled labor from the research sector[29]

$$h_{\mathrm{F}} = \frac{1}{s} \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}. \tag{25}$$

Both the positive effect of adoption on innovation, and the negative effect of the relative cost of skilled workers on innovation, appear in (25). The next proposition formalizes the complementarity between adoption and innovation.

Proposition 4. *An increase in technology adoption increase the measure of frontier technology by a constant elasticity*

$$-\frac{\partial \log A_F}{\partial \log z} = \frac{\lambda}{1-\phi} \frac{\tilde{\rho}}{g_F + \delta_I}.$$

The term $\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}$ characterizes the passthrough from adoption to profits, while the term $\frac{\lambda}{1-\phi}$ characterizes the passthrough from profits to innovation, which depends on the entry technology in (10). Both proposition 3 and 4 are partial equilibrium results, and it remains to be seen how they shake out in general equilibrium.

Imposing market clearing for skilled labor

$$h_{\mathrm{F}} + h_{\mathrm{D}} = h_{\mathrm{tot}},$$

and using (23) and (25) leads to an intuitive expression that highlights the role of the skill premium and technology adoption in balancing the private gains of innovation and adoption across firms and

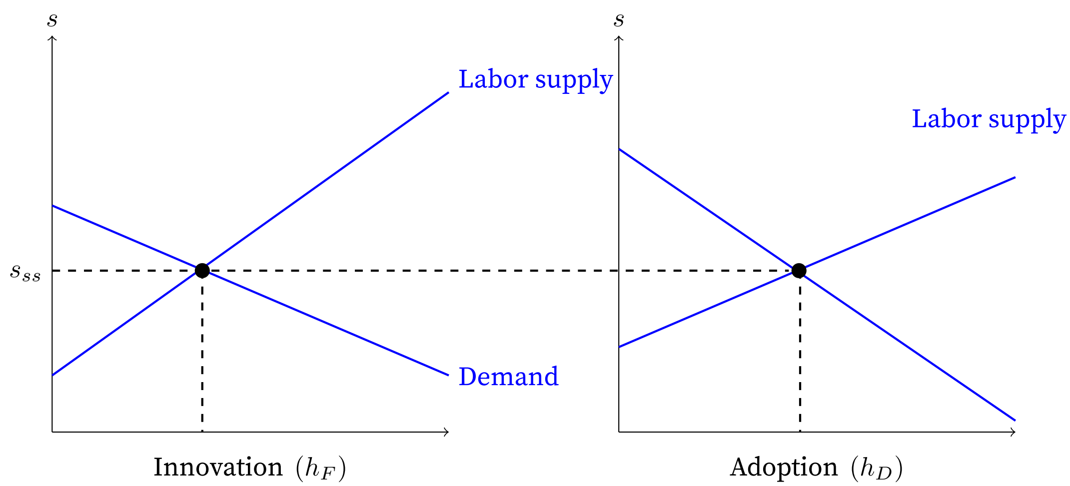
[29] Since $\dot{a}_{\mathrm{F}} = 0$ along a balanced growth path, the resource constraint implies $\frac{\gamma h_{\mathrm{F}}^{\lambda}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}} = a_{\mathrm{F}}$. Inserting this into the free entry condition, $v_{\mathrm{I}} = \frac{s h_{\mathrm{F}}^{1-\lambda}}{\gamma}$, yields normalized demand for skilled labor in the research sector.

sectors

$$\frac{1}{s} z^{-\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}} \Lambda_{\mathrm{F}} + \frac{1}{s} \Lambda_{\mathrm{D}} = h_{\mathrm{tot}}, \tag{26}$$

where Λ_{F} and Λ_{D} only depend on parameters which are constant along a balanced growth path.[30] A simple plot in figure 1 illustrates their interactions. Both adoption activity and innovation activity are downward sloping in the skill premium. While aggregate labor supply is fixed, it is upward sloping for each sector individually and equilibrium is reached when the relative price of skill clears both markets.[31]

Figure 1. Market Clearing for Skilled Labor



Given a solution for the skill premium, the relative share of labor devoted to adoption vis-a-vis innovation in the decentralized allocation reads

$$\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}} = \frac{\beta}{1-\theta} \frac{1}{\alpha} \left[\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + 1 \right] \frac{z^{-\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}}{1 + \frac{\rho+\delta_{\mathrm{X}}}{(g_{\mathrm{F}}+\delta_{\mathrm{I}})(1-\theta)}}. \tag{27}$$

Two aspects of (27) are noteworthy. First, the allocation is unrelated to research productivity γ. Consequently, neither the allocation of skilled labor across sectors nor the skill premium respond to changes in research productivity in the long run. Second, note that a negative shock to the relative supply of skilled labor would reduce both innovation and adoption activity, but the effect on the research sector would be larger due to second round effects: an increase in s first makes the key input,

[30] Throughout the paper I focus on equilibria where h_{tot} is sufficiently scarce so that $s > 1$.

[31] Note that (26) is a non-linear equation because z itself is a function of s, which is the only aspect of the closed-economy model requiring a numerical solution.

skilled labor, more expensive and reduces both activities. Furthermore, a widening technology adoption gap erodes innovators' profits amplifying the negative effects on innovation. Vice versa, an expansion in skilled labor will simultaneously reduce the technology adoption gap, and boost the share of skilled labor devoted to innovation.

To complete the description of the decentralized equilibrium, note that the equilibrium interest rate, $r=\rho+g_{\mathrm{F}}$, determines the price of capital goods, $p_x=\frac{r+\delta_k}{\alpha}$, and the capital-effective-labor ratio

$$\left(\frac{K}{zA_{\mathrm{F}}L_{\mathrm{P}}}\right)=B_k\cdot\left(\frac{\alpha}{\rho+g_{\mathrm{F}}+\delta_k}\cdot\underbrace{\frac{\sigma-1}{\sigma}\cdot\alpha}_{\text{markup distortions}}\right)^{\frac{1}{1-\alpha}},\tag{28}$$

where I used $X=K$, and $B_k=\alpha^{-\frac{\alpha}{1-\alpha}}(1-\alpha)^{-1}$ is a constant.[32] Markups on the price of capital goods and intermediate goods lead to the standard under-accumulation of capital. The production side aggregates up nicely

$$Y=\left(\frac{zA_{\mathrm{F}}L_{\mathrm{P}}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{\alpha}\right)^{\alpha},\tag{29}$$

and real wages are proportional to productivity $w_L=B_w\cdot zA_{\mathrm{F}}$ where B_w picks up constant parameters including markdowns. Moreover, total assets B in the economy consist of physical capital, ownership of production sector firms, and ownership of research firms, i.e., intellectual property

$$B=K+M\cdot V+A\cdot V_{\mathrm{I}}+(A_{\mathrm{F}}-A)\cdot\int_0^{\overline{\tau}}V_{\mathrm{I}}(\tau)\,dF(\tau)$$

where the second term adds the value of firms in the production sector, the third term includes the value of adopted ideas, and the final term adds the value of ideas not yet adopted, taking into account that innovations closer to adoption are more valuable where $\overline{\tau}=\sup\{\tau_j\}_{j\in\Omega_{A_{\mathrm{F}}}}$. I next contrast the decentralized allocation with the planner solution to understand the normative properties of the model.

2.4 Constrained Planner Problem

[33] The allocation of skilled labor will be generically inefficient, and the inefficiency is essential in order to understand the response of the economy to globalization. The only margin the planner decides on is the allocation of skilled labor between innovation and adoption, which means I take the exter-

[32] Using total labor $H+L$ instead of production labor engaged in production L_{P} to define capital intensity only impacts the constant term.

[33] See Klenow and Rodriguez-Clare (2005) for a related discussion of the externalities inherent to a model with innovation and adoption.

nality introduced by endogenous firm entry in the production sector as given. The planner solves

$$\max_{\{H_{\mathrm{F},t},c_t\}_{t\geq 0}} \int_0^{\infty} e^{-\tilde{\rho}t} \log(c_t)\, dt,$$

subject to the following constraints

$$\begin{aligned}
Y &= \left(\frac{AL_{\mathrm{P}}}{1-\alpha}\right)^{1-\alpha} \left(\frac{K}{\alpha}\right)^{\alpha}, \\
\dot{K} &= Y - C - \delta_k K \\
\dot{A}_{\mathrm{F}} &= \gamma A_{\mathrm{F}}^{\phi} L^{1-\lambda} H_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}} A_{\mathrm{F}} \\
\dot{A} &= \nu A^{\theta} A_{\mathrm{F}}^{1-\theta} \left(\frac{H-H_{\mathrm{F}}}{M}\right)^{\beta} - \delta_{\mathrm{I}} A
\end{aligned}$$

and $\frac{\dot{L}}{L} = \frac{\dot{H}}{H} = g_L$.

The constrained efficient allocation of skilled labor across innovation and adoption equals

$$\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{SP}} = \frac{\beta}{1-\theta}\frac{1}{\lambda}\left[\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}} + (1-\phi)\right] \tag{30}$$

where SP stands for social planner. Recall the decentralized allocation (DC)

$$\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{DC}} = \frac{\beta}{1-\theta}\frac{1}{\alpha}\left[\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}} + 1\right]\frac{z^{-\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}}{1+\frac{\rho+\delta_{\mathrm{X}}}{(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}}, \tag{31}$$

which generically differs from the planner solution implying that the allocation of skilled labor to innovation vis-vis adoption activity is inefficient.

This inefficiency is crucial to understanding the impact of market integration in the open economy version of the model as pointed out in Perla, Tonetti, and Waugh (2021). Only if there is insufficient technology adoption to begin with, which is then amplified by specialization in innovation in the open economy, is there a chance for the model to generate sluggish and uneven growth patterns in the aftermath of market integration. I thus provide a comprehensive discussion of the discrepancy between planner and decentralized allocation. To do so, I generalize the baseline mode and assume that the production of intermediate goods also requires skilled labor as a production factor

$$y_i = \left(\int_{j\in\Omega_{A_i}} \left(\frac{x_{ij}}{\alpha}\right)^{\alpha} dj\right)\left(\frac{l_i^{1-\eta} h_{\mathrm{p},i}^{\eta}}{1-\alpha}\right)^{1-\alpha}$$

with $\eta \in [0,1)$, and $h_{\mathrm{P}} := \frac{\int h_{\mathrm{p},i}\, di}{L}$ analogous to the definition of h_{F} and h_{D}. This extension allows me to isolate the spillovers that render innovation and adoption inefficient, as I can shut down each margin separately. Without this additional use for skilled labor, the question is mute since after shutting down the innovation or adoption margin there would only be one use for skilled labor. Planner and decentralized allocation would then trivially agree that all skilled labor should be devoted to whichever activity is left.

Inefficient Adoption. I first turn attention to the adoption margin by assuming that frontier growth is exogenous, which brings the setup close to Parente and Prescott (1994)'s model of technology adoption. The ratio of skilled labor devoted to adoption vis-a-vis production in the decentralized equilibrium equals

$$\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{P}}}\right)^{\mathrm{DC}} = \frac{\beta}{1-\theta}\frac{1}{\eta}\left\{\frac{1}{1+\frac{\rho+\delta_{\mathrm{X}}}{(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}}\right\}. \tag{32}$$

This expression contrasts with the solution to the constrained planner problem in (33)

$$\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{P}}}\right)^{\mathrm{SP}} = \frac{\beta}{1-\theta}\frac{1}{\eta}\left\{\frac{1}{1+\frac{\tilde{\rho}}{(1-\theta)(g_{\mathrm{F}}+\delta_{I})}}\right\}. \tag{33}$$

Since $\tilde{\rho} < \rho + \delta_{\mathrm{X}}$, the decentralized allocation suffers from underinvestment in technology adoption. As argued before, this inefficiency is a generic feature of models where incumbent firms make costly adoption choices, and entrants learn from incumbents. Note that I abstract away from any learning spillovers among incumbents, which would amplify this inefficiency.[34] Moreover, the result highlights how the efficiency result in Parente and Prescott (1994) constitutes a knife-edge case hinging on zero population growth ($g_L = 0$) and no churn among incumbents ($\delta_{\mathrm{X}} = 0$), in which case (32) and (33) agree. If there was firm entry in Parente and Prescott (1994), one would have to assume some form of spillover to tie together the productivity of incumbents and entrants so that a stationary firm size distribution emerges. Vice versa, one can make technology adoption efficient within my framework by setting $\delta_{\mathrm{X}} = -g_L$.[35]

Inefficient Innovation. To isolate the inefficiencies introduced in the research sector I drop the adoption margin and set $z = 1$.[36] The constrained efficient allocation follows

$$\frac{h_{\mathrm{P}}}{h_{\mathrm{F}}} = \frac{\eta}{\lambda}\left\{\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}} + 1 - \phi\right\}, \tag{34}$$

which is virtually identical to the planner solution in Jones (1995). Consequently, the same three sources of inefficiency arise, which I shall mention only briefly: first, there is a dynamic knowledge externality parameterized by ϕ, which could be positive or negative. Second, the instantaneous congestion externality $\lambda < 1$ raises the private returns above the social returns. Lastly, the research

[34] There is a large literature suggesting that technology adoption effort may be inefficiently low, see for example Foster and Rosenzweig (2010) in the context of agricultural production in developing economies.

[35] Instead of negative death shocks, this would then be a model where positive "spin-off" shocks restore efficiency by shutting down entry of new firms not belonging to incumbents.

[36] This model version combines the two-types-of-labor setup in Romer (1990) using Jones (1995)'s semi-endogenous version.

sector's markup $\frac{1}{\alpha}$ leads to incomplete surplus appropriation.

The decentralized allocation in the model without adoption equals

$$\frac{h_{\mathrm{P}}}{h_{\mathrm{F}}} = \frac{\eta}{\alpha}\left(\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}+1\right), \tag{35}$$

which bears out all three externalities. For the innovation-production tradeoff considered in (35), Jones (1995) concludes that for reasonable parameter values, the economy is characterized by insufficient innovation even when there are negative research externalities with $\phi < 0$ and $\lambda < 1$.

Jones' conclusion is in part driven by the assumption that the production sector is efficient. In a model with technology adoption, as I have shown before, this tradeoff becomes more interesting since technology adoption features externalities as well. It is ex ante unclear weather the externalities in innovation vs. technology adoption dominate. I will return to this issue when I quantify the model but the takeaway here is that an endogenous technology adoption margin makes it more likely that there is too much innovation vis-a-vis adoption in contrast to the standard innovation-production tradeoff that the literature has focused on.

A final remark before I conclude this welfare analysis relates to the fact that the full model with both technology adoption and innovation features externalities that are absent when considering the simplified model where I isolated each margin separately. To see this, assume $\alpha = \lambda$, $\phi = 0$, and $\delta_{\mathrm{X}} = -g_L$, which renders innovation and adoption efficient when considered in isolation. In that case, it is still true that planner and decentralized allocation disagree, which can be seen by computing the ratio of the planner and the decentralized allocation

$$\frac{\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{SP}}}{\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{DC}}} = z^{\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}\left(1+\frac{\tilde{\rho}}{(1-\theta)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)}\right). \tag{36}$$

This ratio could be either smaller or larger one, so it is unclear if the decentralized allocation features too little or too much adoption. Because of the advantage of backwardness on the one hand, and the wait-in-line assumption and congestion in bringing ideas to market on the other, the direction of the bias is ambiguous. Neither do firms which adopt technology internalize the positive effect on innovators by shortening the waiting time, nor do innovators internalize their positive effect on adoption operating through the advantage of backwardness.

2.5 Extensions

An important assumption in the baseline model is that the entry cost in the intermediate goods sector is paid in production labor. The downward sloping relationship between the skill premium and the demand for skilled labor for adoption purposes is directly related to the fact that long-run firm profits are proportional to the cost of entry, which in turn is proportional to production worker wages due to free entry. A more general version could use an entry cost based on a Cobb-Douglas bundle of high-skill and production labor, $f_{\mathrm{E}} w_L^{\mu} w_H^{1-\mu}$.[37] In that case the effect of an increase in the price of skill on technology adoption changes to

$$\frac{\partial \log z}{\partial \log s} = \frac{\beta}{1-\theta} \cdot \mu. \tag{37}$$

Clearly, the impact of a rise in the skill premium on technology adoption is weaker for $\mu < 1$. There would, however, be an additional negative effect on firm entry, i.e., $\frac{\partial \log m}{\partial \log s} < 0$ for $\mu < 1$. Since entry costs partially depend on high-skilled wages, a rising skill premium simultaneously reduces firm entry.[38] Additional extensions in the appendix consider heterogenous firms with imperfect knowledge spillover,[39] introducing skilled labor inputs into innovation, endogenizing skilled labor supply,[40] complementary public adoption investment, and skill-biased technological change.

3 Open Economy

In this section, I generalize the setup to a multi-country open economy growth model. Countries differ in two key dimensions, research productivity γ_c and skill endowment $h_{\text{tot},c}$ where $c \in C$ is a country index of a total of N countries. Preferences, the size of production labor across countries L_c,[41] and non-research related technology including technology adoption are identical across countries. I assume countries frictionlessly trade an undifferentiated final good. I abstract away from intermediate goods trade in the production sector.[42] Lastly, I assume that capital goods are produced locally using capital accumulated by the domestic economy, and I impose that the current account is

[37] Assuming entry cost are paid in labor, as opposed to final goods, is essential to obtain a sensible model of long-run firm entry, see Klenow and Li (2024).

[38] I abstract away from this margin here, but it may be of relevance to the literature studying the slowdown of business dynamics, see Decker et al. (2017), Decker et al. (2020), or Karahan, Pugsley, and Şahin (2019). A rising skill premium will negatively affect firm entry whenever firm entry is a relatively skill-intensive activity so the framework might be useful to understand this pattern as well. This point is related to Salgado (2020) where skill-biased technological change leads to less entry into entrepreneurship.

[39] See also related work in progress in Trouvain and Violante (2025), which focuses on investment-specific technological change. In the heterogeneous firm extension, a meaningful distinction between the extensive and intensive margin of technology adoption can be drawn, i.e., whether a specific technology is used in a country, and whether all firms in a country use this specific technology, see Comin and Hobijn (2004) on empirical evidence on this issue. Other extensions include a generalized adoption technology that allows for the government to make complementary investments for technology to be adopted, think of automobiles and the construction of the highway system.

[40] Surely skilled labor supply is somewhat elastic, but not nearly enough to counteract the substantial increase in the skill premium observed over the past couple of decades.

[41] I will allow for differences in country size in the quantitative section. All insights in this section remain intact when allowing for country size heterogeneity but the exposition is slightly less elegant.

[42] Intermediate goods trade a la Krugman (1980) can be added without complication. Exporting neither raises nor reduces normalized profits of firms producing differentiated varieties, so this margin would not interact with the incentives to adopt technology. It would clearly raise the gains from trade.

balanced. I thus abstract away from offshoring,[43] and shut down inter-temporal trade motives. Importantly, even if a domestic idea is embodied in foreign capital abroad, the domestic inventor still receives profits coming from the markup applied to the capital good.

Equilibrium in the Open Economy. I define the world technological frontier as the sum of all distinct ideas in each country, $A_{\mathrm{F}}^{\mathrm{W}} := \sum_c A_{\mathrm{F},c}$. The knowledge spillover A_{F}^{ϕ} is global in the integrated equilibrium,[44] which induces the following law of motion of ideas in economy $k \in C$

$$\dot{A}_{\mathrm{F},k} = \frac{\left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi} H_{\mathrm{F},k}}{f_{\mathrm{R},k}} - \delta_{\mathrm{I}} A_{\mathrm{F},k}, \tag{38}$$

where W denotes world aggregates, and entry is subject to the familiar local externality $f_{\mathrm{R},k} = \frac{\left(\frac{H_{\mathrm{F},k}}{L_k}\right)^{1-\lambda}}{\gamma_k}$.

The net present discounted value of an innovation in the steady state combines profits over all countries, and reads

$$V_{\mathrm{I}} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha L_{\mathrm{P}}}{A_{\mathrm{F}}^{\mathrm{W}}} \sum_c w_c z_c^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} \tag{39}$$

where I used the assumption that countries have the same amount of production labor. In the absence of trade cost the value of an innovation is unrelated to where the idea was developed so there is no country subscript. Consequently, I can use the free entry condition, $V_{\mathrm{I}} = f_{\mathrm{R},k} w_{H,k} \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{-\phi}$, to derive countries' relative research effort as a function of relate skilled worker wages and research productivities

$$\frac{h_{\mathrm{F},k}^{1-\lambda}}{\gamma_k} w_{H,k} = \frac{h_{\mathrm{F},c}^{1-\lambda}}{\gamma_c} w_{H,c} \quad \forall c \tag{40}$$

where I assume $\lambda < 1$ so every country is engaged in some innovation.

Next, using the resource constraint (38), the share $\chi_k := \frac{A_{\mathrm{F},k}}{A_{\mathrm{F}}^{\mathrm{W}}}$ of ideas developed in country k along a balanced growth path reads[45]

$$\chi_k = \frac{\gamma_k h_{\mathrm{F},k}^{\lambda}}{\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) a_{\mathrm{F}}^{\mathrm{W}}} \tag{41}$$

where $a_{\mathrm{F}}^{\mathrm{W}} := \left(\frac{A_{\mathrm{F}}^{\mathrm{W}}}{L}\right)^{1-\phi}$. Combining (41) with (40), and replacing $w_{H,k} = s_k^{\frac{1-\beta-\theta}{1-\theta}} \cdot b_t$, where b_t captures

[43] The production location of capital goods is related to a recent literature on multinational production and offshoring, see for instance Antras, Fort, and Tintelnot (2017) or Arkolakis et al. (2018). Since capital goods are assembled using capital, which in turn is produced using labor, the location of production for capital goods matters for wages and welfare. I avoid this complexity by assuming capital goods are produced locally.

[44] See Grossman and Helpman (1991a) for an in-depth discussion of this issue. Global knowledge spillovers seem a natural assumption in a model of long-run growth.

[45] Note $\dot{A}_{\mathrm{F},k} = \gamma_k \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi} h_{\mathrm{F},k}^{\lambda} L - \delta_{\mathrm{I}} A_{\mathrm{F},k}$ so along a balanced growth path $\frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{\gamma_k} = h_{\mathrm{F},k}^{\lambda} L \frac{A_{\mathrm{F}}^{\mathrm{W}}}{A_{\mathrm{F},k}} \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi-1} \Rightarrow \chi_k \left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) = \frac{\gamma_k h_{\mathrm{F}}^{\lambda}}{a_{\mathrm{F}}^{\mathrm{W}}}$.

long-run wage growth, yields

$$\chi_k = \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}}. \tag{42}$$

Equation (42) expresses the steady state share as a function of exogenous parameters and skill premia, which motivates the following assumption.

Assumption 3. *Assume that* $\beta + \theta < 1$.

This assumption ensures that the share of ideas produced in a country is falling in the skill premium.[46] Once imposed, the model is well behaved. Since the knowledge spillover is global, and the congestion force local, no one country will capture the entire market for ideas.

Demand for skilled labor in innovation follows from combining free entry with the resource constraint, $\frac{\chi_k(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\gamma_k h_{\mathrm{F},k}^{\lambda}} = \frac{1}{a_{\mathrm{F}}^{\mathrm{W}}}$, which, after some simplification, leads to

$$h_{\mathrm{F},k} = \Lambda_{\mathrm{FO}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_c s_c^{-\frac{\beta}{1-\theta}\left(1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}\right)} \tag{43}$$

where Λ_{FO} picks up terms that are constant along a balanced growth path. Combining this with demand for skilled labor in adoption, and imposing labor market clearing, delivers a system of equations that pin down skill premia across countries

$$h_{\mathrm{tot},k} = \frac{\Lambda_{\mathrm{D}}}{s_k} + \frac{\Lambda_{\mathrm{FO}}\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_c s_c^{-\frac{\beta}{1-\theta}\left(1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}\right)}. \tag{44}$$

Given a set of skill premia solving (44), each countries' research share and technology adoption gap follows directly $\{\chi_c, z_c\}_{c\in C}$, as well as the normalized world technology $a_{\mathrm{F}}^{\mathrm{W}} = \frac{\sum_c \gamma_c h_{\mathrm{F},c}^{\lambda}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}$. The rest of the equilibrium description is identical to the closed economy.[47] I next turn to the implications of the theory regarding cross-country income differences and the impact of market integration on growth.

[46] I need to ensure that an increase in the skill premium in some country c leads to an increase in the real wage of skilled workers in country c relative to some alternative country k. If technology adoption is overly sensitive to the skill premium, this need not be true.

[47] To pin down the open economy equilibrium, a simple algorithm based on market clearing in equation (44) works well where the skill premium is raised whenever there is excess demand for skilled labor in a country using

$$s'_k = \frac{\Lambda_{\mathrm{D}}}{h_{\mathrm{tot},k}} + \frac{1}{h_{\mathrm{tot},k}} \frac{\Lambda_{\mathrm{FO}}\gamma^{\frac{1}{1-\lambda}} s_{k,n}^{-\frac{\lambda(1-\theta)-\beta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_{c,n}^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_c s_{c,n}^{-\frac{\beta}{1-\theta}\left(1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}\right)}$$

where the new starting point $s_{k,n+1}$ uses a simple relaxation scheme $s_{k,n+1} = \tilde{\alpha}s'_k + (1-\tilde{\alpha})\, s_{k,n},\ \tilde{\alpha} \in (0,1)$. I verify that this procedure converges to the same solution for different starting values. I am not aware of a general uniqueness proof for the multi-country version of this model. After inspecting the market clearing condition in (44), the reader will note that neither the gross substitutes property used in Alvarez and Lucas (2007), nor uniqueness proofs based on non-homogenous integral equations as in Allen and Arkolakis (2014) apply.

Cross-Country Inequality. Embedding a skill-intensive technology adoption choice into an otherwise standard semi-endogenous growth model delivers a coherent theory of cross-country inequality and growth. First, countries that are part of the global economy adopted technology from the world technological frontier, $A_{\mathrm{F}}^{\mathrm{W}}$, while their relative productivity is tied to the amount of skilled labor devoted to technology adoption, $z_k \propto h_{\mathrm{D},k}^{\frac{\beta}{1-\theta}}$. Countries thus grow at the same long-run rate and income differences are level effects, consistent with the data (Parente and Prescott, 1993; Jones, 2016). These income differences would be accounted for by differences in TFP in line with the literature on development accounting (Klenow and Rodriguez-Clare, 1997; Caselli, 2005).[48] Moreover, TFP gaps are driven by countries' lack of technology adoption, which in turn is driven by low skilled labor endowment. This is consistent with the tight correlation between TFP and skilled labor endowments in the data, and reconciles Mankiw, Romer, and Weil (1992) focus on human capital with Parente and Prescott (1994) emphasis on TFP and technology adoption.

Second, note that the adoption margin helps reconcile scale effects inherent to idea-based growth models with the seeming lack of scale effects in the data. Scale effects here concern the counterfactual prediction that larger countries enjoy a higher GDP per capita.[49] This is true in most growth (and international trade) models whenever ideas are a non-rival good. It is easy to see that a country's productivity is detached from its size precisely because countries adopt technology developed elsewhere.

The idea that technology adoption resolve the issue of scale effects is not new, see Klenow and Rodriguez-Clare (2005). The contribution here is to provide a micro-founded model of firm entry and adoption that makes precise what assumptions are needed to develop a theory consistent with the lack of scale effects in the cross-section of countries.[50] The key is to construct a technology adoption margin that features a constant-returns to scale property when aggregated. Only when firm entry cost are paid in production labor, and technology adoption uses high-skilled labor will the productivity of a country be detached from its size whilst tightly correlated with its skilled labor share. To understand why this is the case, take account of the firm entry margin. In a more populous economy there are more firms in the production sector. Since the adoption gap is a function of the number of skilled workers per firm, a larger economy needs more skilled labor to achieve the same adoption gap as a smaller one. This extensive margin effect leads to a model where the ratio of skilled labor –and not the level– is the key determinant of a country's position in the world. This justifies the assumption that firm entry in the production sector is intensive in production labor.[51] Scale effects do, of course,

[48] Measuring productivity in the open economy is difficult due to the role of international trade and price effects, see Burstein and Cravino (2015). Here, I mean productivity as the rate at which production sector firms can turn raw capital and labor into differentiated varieties. This may not coincide with productivity measured on the country-level due to the confounding role of innovation and skill prices. It would, however, be the relevant metric to determine production worker wages in each country.

[49] See Alesina, Spolaore, and Wacziarg (2005) for a summary of empirical work on the issue, and Ramondo, Rodríguez-Clare, and Saborío-Rodríguez (2016) for recent work in international trade.

[50] See Gross and Klein (2024) for related work that deals with the issue of cross-country scale effects by differentiating between global and local ideas.

[51] If firm entry in the production sector was skill intensive, the model would imply that there are relatively fewer firms in

matter in innovation, and the theory is consistent with the empirical fact that most innovation occurs in large, skill-rich countries.[52]

Cross-Country Skill Premia. One serious empirical challenge to models which elevate the role of skilled labor in explaining cross-country inequality is the counterfactual link between income per capita and skill premia. If skill is key, and skill is scarce in poor countries, it ought to be true that the price of skill is high in poorer countries whenever skill groups are imperfect substitutes for another, see Caselli and Coleman (2006). The weak empirical relationship between skill premia and output per capita then casts doubt on the elevated role of skilled labor espoused here.

The model addresses this puzzle to some extent. A classic insight from international trade theory (Stolper and Samuelson, 1941) is that factor price equalization forces can attenuate the link between skill premium and skill scarcity when innovation is skill-intensive. Skill-rich countries could specialize in innovation sustaining a high skill premium despite vastly greater skilled labor endowments relative to poor countries. In addition to this standard explanations, a model with skill-intensive endogenous technology adoption choice offers a novel perspective on how to reconcile relatively low skill premia in poor economies with the role of skilled labor in development once one additional assumption is made: suppose it was difficult for production sector firms in developing economies to scale up consistent with the literature on misallocation and development (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Hsieh and Klenow, 2014). If so, adopting superior technology is less beneficial since the firm's technological edge won't translate into a larger operation and increasing profits as it would in a frictionless benchmark. This would not only suppress technology adoption but simultaneously reduce demand for skilled labor, which in turn induces a reduction in the skill premium in general equilibrium. I do not focus on this margin in this paper, which deserves its on thorough investigation, but I will introduce a simple firm reallocation friction in the East in the autarky equilibrium to match low skill premia observed in the data.[53]

Patterns of Trade and Impact of Market Integration. Trade is unbalanced. Countries more specialized in innovation earn royalties abroad, matched by final goods flows from the net importers of technology

$$net\_trade\_inflow_k = \frac{\sigma - 1}{\sigma} \alpha \left(1 - \alpha\right) Y^{\mathrm{W}} \left[\chi_k - \frac{Y_k}{Y^{\mathrm{W}}} \right]$$

where $Y^{\mathrm{W}} = \sum Y_c$ is world GDP. The patterns of net trade depend on the vector of fundamental research productivity $\{\gamma_c\}$ and skill endowment $\{h_{\mathrm{tot},c}\}$, allowing for both Ricardian technology differences and Heckscher-Ohlin factor endowment forces. In an equilibrium where technology is largely produced in the West due to higher research productivity or greater skill endowments, final goods

poor countries, which is not true. If unskilled labor could be used for technology adoption, there would be no link between skill endowments and productivity, and additional wedges as in Parente and Prescott (1994) would be required to generate cross-country inequality.

[52] The role of endogenous entry in dealing with scale effects is reminiscent of Young (1998).

[53] This would then also prevent skilled workers in advanced economies from wanting to migrate to poor countries, which is an odd but natural implication of a general equilibrium model where skill matters.

flow from East to West as compensation for technology usage.

Two points are noteworthy about the relationship between international trade and growth. First, in this simple model the trade elasticity of the final good is assumed infinite, which implies that final goods trade is not important in itself. Yet, final goods trade is key in order to access foreign technology: if there was no trade, it would not be possible to compensate owners of technology across borders.[54] Second, gains from integration can be large even when trade flows are relatively small because access to ideas doesn't require (at least in this model) that the entire capital good is shipped across countries. It suffices to pay a royalty to the foreign country, which constitutes only a fraction of the value of the capital good.[55]

The model admits a simple sufficient statistic for the log-run wage effects of market integration, similar to Arkolakis, Costinot, and Rodríguez-Clare (2012).

Proposition 5. *The ratio of production worker wages in the open economy vs. closed economy for some country k reads*

$$\frac{w_k^{open}}{w_k^{closed}} = \underbrace{\left(\frac{h_{F,k}^{open}}{h_{F,k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}}}_{\text{Innovation margin}} \underbrace{\left(\frac{s_k^{open}}{s_k^{closed}}\right)^{-\frac{\beta}{1-\theta}}}_{\text{Adoption margin}}, \tag{45}$$

where I netted out long-run growth. The effect on skilled worker wages follows $\frac{w_H^{open}}{w_H^{closed}} = \frac{w^{open}}{w^{closed}} \cdot \frac{s^{open}}{s^{closed}}$. The key distinction to Arkolakis, Costinot, and Rodríguez-Clare (2012) is that the change of the skill premium is part of the sufficient statistic.[56]

The impact of market integration on the technological frontier are captured in increasing innovative effort in the home economy $\left(\frac{h_{F,k}^{open}}{h_{F,k}^{closed}}\right)^{\frac{\lambda}{1-\phi}}$ and gains from ideas developed in other countries, $\left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}}$, which depend on a constant scale elasticity $\frac{1}{1-\phi}$ as well as the share of ideas developed in the home economy. If this share is small, the gains are large. The logic is the same as in ACR where a large import share suggest a country has much to lose if it fell back to autarky. The novel feature in (45) is the endogenous adoption margin which shows up in the skill price ratio $\left(\frac{s^{open}}{s^{closed}}\right)^{-\frac{\beta}{1-\theta}}$. An increase in the skill price ratio, ceteris paribus, hurts production workers. The reason is that a rising skill premium leads to less domestic technology adoption, which allows for a richer response of market integration on growth. In particular, the effects are now dependent on whether the country in-

[54]The way the model is set up, foreign technology owners don't care about how large trade costs – they gain from foreign adoption no matter the size of the trade cost. If there were even small fixed cost paid by the owners of technology to set up shop abroad, they would become sensitive to the size of trade cost, and the size of the market. See Burstein and Monge-Naranjo (2009) or McGrattan and Prescott (2010) for related frameworks where final goods are undifferentiated and traded is required to compensate owners of technology

[55]A country in my baseline calibration would have to export 14% of its final output when its entire idea stock is held by foreigners. In contrast, in a symmetric equilibrium where every country holds an even share of global technology with sufficiently rich financial markets, no goods would have to be shipped whatsoever as net trade flows are zero and trade in financial asset would make trade in final goods redundant.

[56]The knowledge spillover is assumed to be local in the autarky equilibrium.

tegrates with similar or very different trading partners, which for the purposes of this paper concerns heterogeneity emanating from research productivity and relative skill endowments, $\{\gamma_k, h_{\text{tot},k}\}$.

Consider first integration between symmetric countries with $\gamma_k = \gamma$, $h_{\text{tot},k} = h_{\text{tot}} \,\forall k$. This benign scenario delivers the standard variety gains from trade without negative distributional effects, and no changes in the adoption gap, summarized in the following proposition.

Proposition 6. *Symmetric integration leaves the skill premium and the technology adoption gap unchanged relative to autarky, but induces gains from integration by a factor of* $N^{\frac{1}{1-\phi}}$.

The proposition follows from noting that for symmetric countries the open economy market clearing condition in (44) collapses to the closed economy. Consequently, skill premium and adoption gap don't respond in the long run. By symmetry, every country produces a fraction $\frac{1}{N}$ of ideas, and plugging this into (45) completes the proof.[57] In the case of symmetric integration, real wage gains accrue to all workers without adverse inequality effects. The strength of this response depends on ϕ, i.e., how much harder ideas are to find. This result contrasts with the case of asymmetric integration described next.

In the case of asymmetric integration, the impact of market integration is ambiguous. To see this, consider a two-country case where all ideas in the integrated equilibrium are produced in the advanced home economy so $\chi \approx 1$. The modified present discounted value of an innovation in the open economy equals

$$V_{\text{I}} = \underbrace{\left(\frac{1}{\tilde{\rho} + g_{\text{F}} + \delta_{\text{I}}}\right) \frac{\alpha L_{\text{P}} w}{A_{\text{F}}} z^{\frac{\tilde{\rho}}{g_A + \delta_{\text{I}}}}}_{\text{same as closed economy}} \left\{ 1 + \underbrace{\frac{L_{\text{P}}^* w^*}{L_{\text{P}} w} \left(\frac{z^*}{z}\right)^{\frac{\tilde{\rho}}{g_A + \delta_{\text{I}}}}}_{\text{additional market size effect}} \right\}. \tag{46}$$

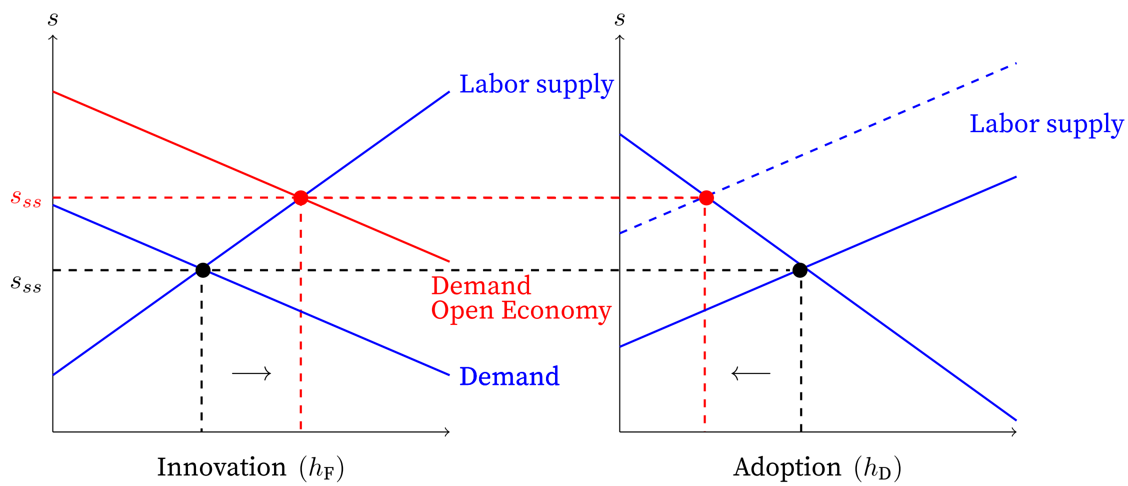
where profits now accrue both at home and abroad. Equation (46) reveals that the strength of the foreign idea demand shock depends on i) the adoption gap $(z^*)^{\frac{\tilde{\rho}}{g_A + \delta_{\text{I}}}}$ in the emerging market, as well as ii) GDP summarized in $L_{\text{P}}^* w^*$ relative to variables in the advanced economy. Market integration directly increases the market size of innovators, which raises profits that are arbitraged away by increasing entry into innovation. Moreover, convergence in the emerging market ($z^* \uparrow$) would further raise the returns to innovation both because the foreign wage rate increase and because the time it takes for innovation to be adopted abroad is falling. In a model where technology is endogenous, fast adoption in emerging markets and rising returns to innovation in advanced economies are two sides of the same coin.

Rising returns to innovation in the open economy in general equilibrium bring about an reallocation of skilled labor from domestic adoption towards global innovation accompanied by elevated

[57] This result is reminiscent of Krugman (1980) where trade integration induces variety gains but leaves the measure of varieties in each country unchanged. This result relies on the homogenous firm model, and constant markups.

technological frontier growth and an overall increase in the price of skilled labor. At the same time, technology adoption recedes following almost trivially from the factor market clearing condition. While this raises skilled wages, the impact on production workers and the economy as a whole is ambiguous, and depends on whether technological frontier growth dominates the negative effect of weakened technology adoption. Figure 2 summarizes the main argument of this paper graphically.

Figure 2. Open Economy Market Clearing for Skilled Labor



Which effect dominates is a quantitative question, with interest transition dynamics off the steady state. Before I calibrate the model in the next section to study this question, it is worthwhile to discuss why the model allows for a more flexible response of market integration on growth than common trade models, which allow for unequal effects across worker groups but tend to maintain that there are aggregate gains.[58] The key difference is that much of the trade literature operates in efficient environments,[59] which is not the case here.

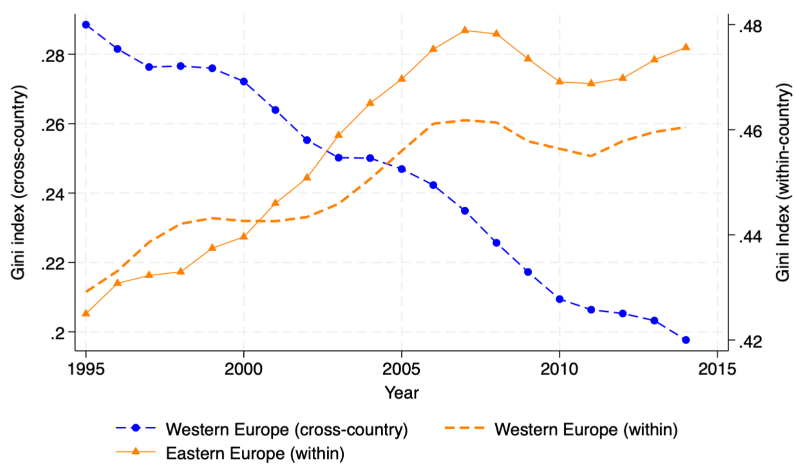
[58] See Wood (1994), Leamer (1994), Feenstra and Hanson (1996) and more recently Galle, Rodríguez-Clare, and Yi (2017) and Adao et al. (2020) for works on trade and inequality. Relatedly, a literature focused on firm heterogeneity, inequality, and globalization has studied the impact of trade on inequality, see Egger and Kreickemeier (2009), Helpman, Itskhoki, and Redding (2010), Liu and Trefler (2008), Sampson (2014), or Burstein and Vogel (2017). Helpman (2016) argues that the overall impact of trade on inequality viewed through the lens of the aforementioned literature appears small.

[59] Atkeson and Burstein (2010), as discussed in Perla, Tonetti, and Waugh (2021), is a particularly illuminating example for the role of efficiency on the impact of market integration on growth.

4 Quantification

I quantify a version of the model to explain three striking features of global growth since the mid 90s, two of which are depicted in figure 3. First, strong growth in emerging markets, especially Eastern Europe and China, have lead to a decline in global inequality measured as dispersion in income per capita across countries. Using PPP-adjusted GDP per capita form the PWT, the cross-country Gini index declines by around seven pp. from 1995 to 2015 when focusing on a set of West and East European economies.[60]

Figure 3. Cross-Country Convergence and Within-Country Divergence



The data is based on the World Inequality Database, see Alvaredo et al. (2020). The gini index is computed over the whole population and uses pre-tax income, split concept. Aggregates are employment-weighted averages within each country group, where I compute the within-country Gini index using a 3-year moving average. Country income is based on ppp-adjusted output per capita using PWT V10. Country list for Western Europe: Austria, Belgium, Denmark, Finland, France, Germany, Iceland, Luxembourg, Netherlands, Norway, Sweden, Switzerland, United Kingdom; Country list for Eastern Europe: Albania, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Montenegro, Poland, Romania, Serbia, Slovakia, Slovenia. I do not consider Southern European countries.

Second, within-country income inequality went up substantially. When measured in terms of pre-tax per capita income the rise in within-country inequality matches the decline in cross-country inequality. Third, while per capita growth was exceptional in Eastern Europe and some parts of Asia, per capita growth in the West was abysmal during this episode of rising market integration. This has given rise to a large literature on the productivity slowdown. The model constructed here can account for to all three facts jointly.

[60] Convergence is not uniform, and many countries, especially in Africa, have seen much less catchup growth, if at all. See Milanovic (2016) for an in-depth discussion of global convergence. The focus on Europe is simply due to easily comparable data series.

To quantify the aggregate implications of the theory I have to pin down a set of parameters $\Theta = \{\rho, \alpha, \delta_k, \sigma, f_{\mathrm{E}}, \delta_{\mathrm{X}}, \{L_c\}, \{\mu_c\}, \{\gamma_c\}, \delta_{\mathrm{I}}, \nu, \beta, \theta, \lambda, \{h_{\mathrm{tot},c}\}, g_L\}$ where the inner parentheses and subscript c indicate that the parameter needs to be set separately for each country, where I now allow for differences in country size L_c. I will offer a straightforward calibration strategy that serves as first pass to study transition dynamics in a complex dynamic environment and illustrate the strength of the key mechanism.

Some of the parameters of the model require micro-data, and I use easily accessible administrative data on firm and employment dynamics in Germany provided by the IAB. The German economy provides a good case study as it experiences a major unanticipated globalization shock as Eastern European economies open up after the fall of the Iron Curtain. The mechanism I study in this paper is more broadly relevant to advanced economies as a whole.

Externally set parameters. I set the capital share equal to .4, capital depreciation equals 5%, and the discount rate is 5%, implying a long-run real rate of 6% roughly consistent with stock market returns. I set the elasticity of substitution across varieties equal to 2.5, consistent with evidence from Broda and Weinstein (2006). I set firm exit rates in the production and research sector equal to 4%, i.e., $\delta_{\mathrm{X}} = \delta_{\mathrm{I}} = .04$. These numbers line up well with German establishment micro data,[61] where I split establishments into research-intensive and production-intensive firms consistent with the two-sector structure of the model.[62]

Adoption. The implications of the theory hinge on the importance of technology adoption, which in turn is directly related to the ratio $\frac{\beta}{1-\theta}$. Cross-country inequality and growth patterns are directly related to the ratio $\frac{\beta}{1-\theta}$, since real wage differences for production workers across countries in the steady state read

$$\frac{w_c}{w_k} = \left(\frac{h_{\mathrm{D},c}}{h_{\mathrm{D},k}}\right)^{\frac{\beta}{1-\theta}} . \tag{47}$$

Conditional on a distribution of the relative amount of skilled labor devoted to adoption across countries $\{h_{\mathrm{D},c}\}$, the parameters $\{\theta, \beta\}$ translate this initial distribution into observed cross country inequality.

Taking logs of (47) and adding a measurement error u allows me to back out β by running the

61

Details on the German micro data and more careful empirical analysis can be found in the Job Market Paper version of this paper, see Trouvain (2023). None of the empirical patterns are controversial, or unique to Germany.

62

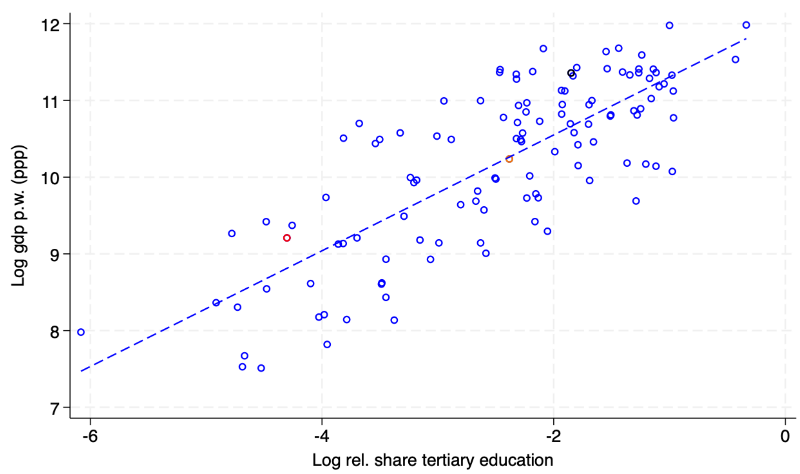
Recall that the higher the arrival rate of death shocks in the production sector, the stronger is the adoption externality. I thus choose conservative values that are lower than the empirical estimates from Garcia-Macia, Hsieh, and Klenow (2019) (ranging from 8% to 4% depending on firm age and time period) or Peters and Walsh (2019) (5.4%) for the US economy. In a related model Sampson (2023) uses a firm exit rate of 10%. Subtle differences between firm vs. product/establishment and unweighted vs. employment weighted vs. revenue weighted measures of exit explain differences in exit rates.

following regression

$$\log z_{c,t} \quad = \quad \alpha + \delta_t + \tfrac{\beta}{1-\theta} \log h_{\mathrm{D},c,t} + u_{c,t}. \tag{48}$$

The slope coefficient through the lens of the model equals $\frac{\beta}{1-\theta}$ where I proxy for production worker wages using GDP per capita and I proxy for h_{D} using the relative share of college-educated workers in each country, i.e. h_{tot}. To this end, I combine data from Barro and Lee (2013) with the PWT and run the regression for the year 2015 to capture the post-integration steady state where more countries have moved toward a market-based open economy.[63] Figure 4 shows a simple cross-sectional plot between

Figure 4. Relative High-Skill Share and Output p.c.



The plot combines ppp-adjusted output per worker from PWT 9.1 with the Barro and Lee (2013) dataset on schooling for a cross-section of countries in 2015. I equate high-skilled labor with the share of completed tertiary education of the population, and low-skilled labor is 1 minus this share. The colored dots refer to Congo, Brazil, and Germany in this order from left to right.

the relative share of skilled labor and output per worker, with an R-square of .55. Note, however, that the mapping between skill share and output per worker is confounded when skilled labor is also devoted to innovation. I thus run a regression where I drop countries above the 90th percentile in terms of output per worker, which account for most innovative effort in the world. Moreover, I use the lagged skill ratio as an instrument in a simple IV regression to deal with measurement error. The resulting regression coefficient equals 0.74 with robust standard error 0.07 .

This identifies the ratio $\frac{\beta}{1-\theta}$, but I need to pin down each parameter individually. I build on the large cross-country growth literature, especially Barro (1991), and chose θ such that the model is consistent with measured catch-up growth across countries. In particular, Barro's "Iron law" (Barro, 1991) suggests countries converge at a rate of 2%, i.e., the coefficient in the cross-country convergence re-

[63]In a closed economy, the logic of the model does not work since country-specific technological frontiers would confound the link between relative skill endowments and real wages.

gression, after controlling for a number of covariates and in particular human capital, is close to $-.02$. I linearize the law of motion of z to show that θ is the key parameter governing this speed of convergence

$$\frac{\dot{z}}{z} \approx \underbrace{(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}_{=\hat{\beta}_B}(\log z_{ss}-\log z_t)+\beta(g_{\mathrm{F}}+\delta_{\mathrm{I}})(\log h_{ss}-\log h_t).$$

If $\delta_{\mathrm{I}}+g_{\mathrm{F}}=5\%$ were to be the case, a reasonable estimate for θ is 0.55 which ensures that $\hat{\beta}_B \approx -.02$. This is consistent with Lucas (2009a)'s calibration of the same advantage of backwardness parameter. In that case, $\beta = .35 \approx \frac{45}{100} * \frac{3}{4}$. Of course, the previous estimate is contingent on the frontier growth rate, which I assume is 1%, and discussed next.

Innovation. The long-run frontier growth rate is endogenous, and equals $\frac{1}{1-\phi}g_L$. To keep the model simple, I will assume a constant long-run growth rate $g_L = 2\%$ with fixed high-skill vs. low-skill shares.[64]

I assume a dynamic knowledge externality of $\phi=-1$ in the baseline calibration. This parameterization is more optimistic than what the evidence in Bloom et al. (2020) suggests, and using a more negative value will lead to a more painful tradeoff between innovation and adoption. Two aspects are noteworthy. First, because there is an endogenous technology adoption gap, the mapping between research effort and productivity is more complicated than in Bloom et al. (2020). In fact, technology adoption represents an omitted variable in their framework through the lens of the theory. Second, the sluggish response of overall GDP growth in advanced economies makes it unlikely that this parameters is large and positive. I will show that calibrating the model using a more optimistic dynamic externality leads to counterfactually strong growth for all groups.

I also need to calibrate λ, which does not find an antecedent in the literature.[65] I proceed as follows. Intuitively, cross-country specialization in innovation for similar countries should contain information on this parameter. If research productivity among innovating countries was identical, $\gamma_c=\gamma$, the model would imply a log-linear relationship between the share of ideas χ, the total number of researchers H_{F}, and country size measured in terms of the production labor force L in steady state

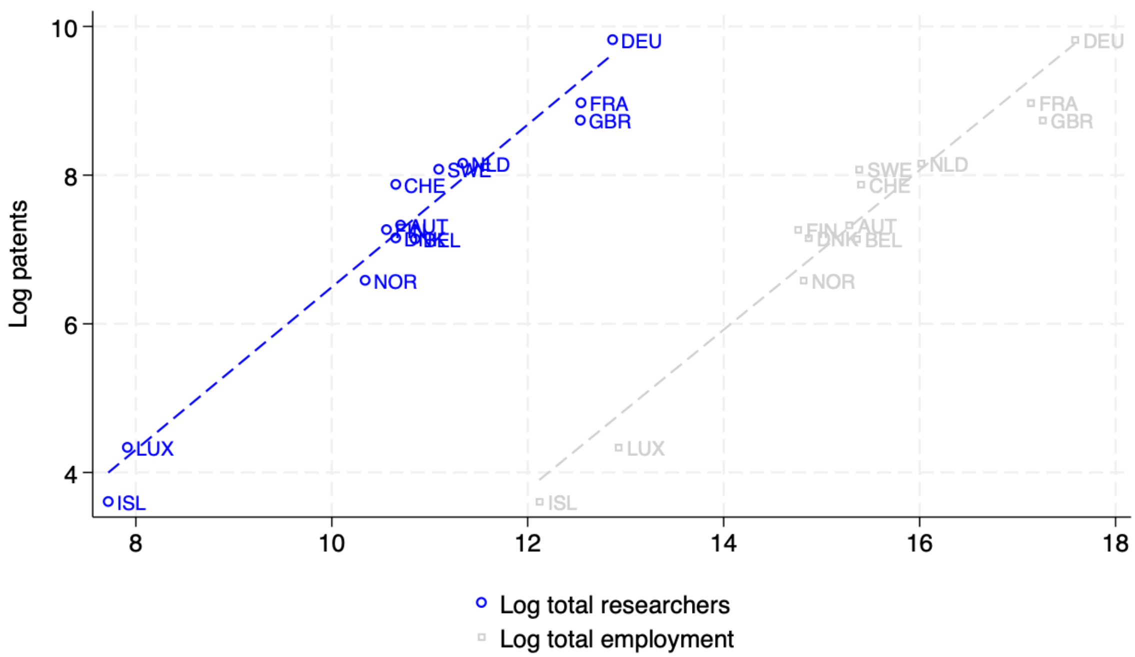
$$\log \chi_c = \alpha_0 + \lambda \log H_{\mathrm{F},c} + (1-\lambda)\log L_c. \tag{49}$$

[64]Slowing population growth and rising educational attainment represent two confounding factors that push in opposite direction: slowing population growth reduces long-run growth in idea-based growth models (Jones, 1995) while rising educational attainment is pushing the other way. The model developed here could in principled be used to study this tradeoff, but one would have to think harder about selection into schooling, and the substitutability between skilled and production workers in production vs. technology adoption vs. innovation. These are important issues I abstract from.

[65]In an earlier version I used Jones (1995)'s original setting with $f_R \propto H_{\mathrm{F}}^{1-\lambda}A_{\mathrm{F}}^{\phi}$ vs. $f_R \propto \left(\frac{H_{\mathrm{F}}}{L}\right)^{1-\lambda}A_{\mathrm{F}}^{\phi}$ in the setup at hand where λ is interpreted as static congestion force. The original setup works just as well for symmetric countries. If countries have different population sizes, however, and the static congestion is local (which is what one would like for a well-behaved open economy equilibrium), one can show that this would induce a strong counterfactual bias toward innovation in small countries. There is another subtle distinction in that the growth rate in Jones (1995) is $g_{\mathrm{F}}=\frac{\lambda}{1-\phi}g_L$ while the long-run growth rate in my model is $g_{\mathrm{F}}=\frac{1}{1-\phi}g_L$.

I assume that research productivity is equal across advanced economies,[66] and I approximate the share of ideas from country c crudely using average internationally protected patents from country c over the period 2011–2019.[67] Combining these assumptions with data on the total number of researchers and total employment from the OECD "Science, Technology, and Innovation" database allows me to estimate version of the (49) to obtain estimates for λ. The fit between total number of researchers and patenting is almost perfect with an elasticity of roughly unity, and a correlation of .97, see figure 5.[68]

Figure 5. Specialization in Innovation



The plot uses data from the OECD "science, technology, and innovation" database. I plot patents flied under the PCT (global patents) against and the total number of researchers similar to Bloom et al. (2020). The set of countries is the same as in the previous plot for the West. Each dot represents a simple average within each country over the period 2011-2019 to avoid the confounding influence of the financial crisis and the pandemic.

Running cross-sectional regressions based on (49) with both total employment and number of

[66] While Sampson (2023) provides evidence that sectoral research productivity differs across countries, he finds that differences are much larger across emerging vis-a-vis advanced economies, and his focus rests on sectoral heterogeneity. It seems reasonable that sectoral heterogeneity overstate differences in research productivity relative to an aggregate country-specific research productivity. I thus view his results as broadly consistent with the spirit of my exercise.

[67] There is clearly an issue of flow vs. stock, which disappears in steady state.

[68] The fact that global patents are tightly related to a country's number of researchers, which, among rich countries, is tightly correlated with country size, highlights once more why a multi-country semi-endogenous growth model is the right model to study long-run growth. Scale matters, and scale differs across countries.

researchers is challenging due to high multicollinearity. Nonetheless, table 17, and especially column 4, suggest a reasonable value of λ around .98. Since small differences in research productivity and skill endowments are likely to induce an upward bias, I make a back of the envelope adjustment and use $\lambda = .9$ as my baseline value. Additional details (and shortcomings of this approach) are discussed in the appendix.[69]

I am left with the following parameters $\{f_{\mathrm{E}}, \{L_c\}, \{\mu_c\}, \{\gamma_c\}, \nu, \{h_{\text{tot},c}\}\}$. I assume that h_{tot} in the advanced economy is .14, roughly consistent with data from Barro and Lee (2013) over the period 1980 – 2015, and similar to the high skill-low-skill ratio in Acemoglu et al. (2018). I target a skill premium of 1.6 in the initial equilibrium for the advanced economy based on Buera et al. (2022), and I target a relative productivity level of $z = .75$ in autarky, which means that the waiting time for an idea to be adopted equals 5.7 years.[70] I use these two variables to pin down the fixed cost of entry in the production sector as well as the shifter in the adoption technology $\{f_{\mathrm{E}}, \nu\} = \{1.25, .21\}$. I normalize production labor in the advanced economy to unity, and assume that the foreign economy is of relative size $\frac{L^*}{L} = .66$.[71] Lastly, I normalize research productivity in the West to unity, $\gamma = 1$, which is without loss of generality.

I will consider different values for γ^* and h^*_{tot} in the quantitative exercise. For the case of asymmetric integration, I assume $\gamma^* \to 0$, $h^*_{\text{tot}} = .05$, which implies that all innovation is produced in the West. I view this as a central feature of market integration in the 1990s and 2000s, see for instance the OECD study by Khan and Dernis (2006) which documents a large increase in patenting in Europe during this period, but with almost no patenting activity in Eastern Europe.[72] In an alternative scenario, I will assume skill endowment and research productivity in the East converge to the values in the West. Before I turn to the quantitative exercise, I have to take a stance on the initial equilibrium.

Initial Equilibrium. The initial autarky equilibrium should be consistent with the wage gap across East and West, and the skill premium within each block. Having close to zero research productivity, however, would imply a counterfactually large wage gap. I fix this by assuming that the East's innova-

[69] Interestingly, the data do not display a home-market effect in the sense that larger countries among this set of advanced economies specialize disproportionately into innovation, see Venables (1987) and more recently Arkolakis et al. (2018). Running a regression of the share of researchers on total or production worker employment delivers a surprisingly precise zero effect.

[70] Since I don't have firm heterogeneity, the reader should think of this wait time as the moment when the product has reached substantial market penetration, and not the first time the capital good is used somewhere in the economy. The distinction is explicit in the heterogeneous firm version of Trouvain and Violante (2025).

[71] In principal, considering the size of Eastern Europe and Asia, this number may seem surprisingly small. Note, however, that in the two country model I have to lump together countries like Poland and China, which have very different income levels. In this calibration workers in the East will produced 2/5th of total output as production worker wages of the two blocks converge in the long run. I abstract away from trade cost, and potential bargaining over innovator rents, which would be important extensions and allow me to consider a relatively larger East. I show in the appendix how to generalize the model to include such features. Note that since the East exerts pull on innovation in the West, the size of the East matters quantitatively, and the larger the East, the stronger are the uneven effects of market integration that the model generates.

[72] The contribution of Eastern Europe at the time is so small that it ends up in a residual category. For more recent years, this assumption may be less appropriate, especially with respect to other emerging markets such like China. Bergeaud and Verluise (2022) provide evidence from patent data suggesting that China is contributing as much as the USA to the technological frontier in recent years.

tion in autarky is easier as it copies technology already invented in the West.[73] Moreover, since skill is relatively scarce in the East, ceteris paribus, the skill premium should be relatively high. In the data, this is not true and I calibrate the model such that the skill premium in East and West are the same in autarky. To achieve this, I introduce a technology adoption friction similar to Parente and Prescott (1994). Specifically, suppose that there is a market-share reallocation friction consistent with empirical findings in Hsieh and Klenow (2014) and parameterized by some $\mu_{\mathrm{D}} < 1$ such that the impact of technology adoption on profits is suppressed

$$\frac{\partial \pi^*_{\text{autarky}}}{\partial z} = \underbrace{\mu_{\mathrm{D}}}_{<1} (\sigma - 1)(1 - \alpha),$$

where $\frac{\partial \pi}{\partial z} = (\sigma - 1)(1 - \alpha)$ is the frictionless benchmark that holds in the West. Intuitively, if the market reallocation friction is such that firms' sales shares were fixed, there would be no incentive to adopt technology. This suppresses the demand for skilled labor twofold. Fist, directly through lower demand for skilled labor in the production sector, and then indirectly through the equilibrium effect of weak technology adoption on the present discounted value of innovation. Similarly, by taxing innovators profits at rate μ_{F}, I could push down demand for skill in the research sector. Consequently, for the right "wedges" any allocation of skilled labor becomes feasible, and the skill premium can be arbitrarily depressed. Since misallocation in developing economies appears to be large (Hsieh and Klenow, 2009), this provides a promising candidate explanation for the weak link between skill scarcity and skill premium. Indeed, Brainerd (1998) summarizes evidence of low levels of inequality within and across worker groups in the Soviet Union, and I loosely follow this evidence by setting the autarky skill premium in the East to the same level as in the West.

Lastly, in a world where ideas are harder to find, it is natural to assume that countries develop the same technology in autarky. While there is no incentive to develop the same idea twice in the integrated equilibrium, varieties could overlap initially so I need to specify which country holds the patent to which varieties right after market integration. I assume that the West holds a share ζ of technology while the East holds $1 - \zeta$.[74] I discipline this initial process of ownership reallocation in

[73] The problem here is that the emerging market block I consider is small with a relative size of .39, and the initial income gap, while large, is not nearly large enough. The model could be applied without this ad-hoc assumption to China, which is a much larger economy starting out from a lower income level. Because the allocation of skilled labor across sectors is unrelated to research productivity in autarky, I simply solve the for allocation in the emerging market using the standard solution routine, and then I scale wages by some constant factor, which is meant to capture that copying is easier than innovating. Note that no matter how easy copying is, as long there is some fixed cost associated with it, there is no incentive to do so in the integrated equilibrium for Bertrand competition would drive profits to zero.

[74] A micro-foundation can be readily provided. All ideas only invented by the West are held by the West. Ideas that overlap in the interval $[0, A^*_{\mathrm{F}}]$ are split across East and West, which can be micro-founded as follows. Suppose that the same idea across countries differs by some quality-metric $q > 0$ such that the effective quality is $q_c = 1 + \epsilon_c + u_c$. The parameter $\epsilon_c > 0$ is a country-specific quality shifter. Let the shock u be bounded between $[-1, 1]$, mean-zero, and with a vanishingly small variance. Further, suppose that an innovator needs to pay a fixed cost f_o for as long as the idea exists. As argued in Acemoglu et al. (2018), such a setup converges in the limit to the simple baseline monopoly pricing when $f_o \to 0, \mathbb{V}[u] \to 0$, and $\epsilon_c \to 0\,\forall c$, but I assume ratio $\frac{\epsilon_c}{\epsilon_k} \to b > 0$ converges to something bounded away from zero at just the right rate. This sustains an ownership

the integrated equilibrium with the drop in income in the East right after the fall of the Iron Curtain, which was substantial, and propose a value of ζ =.99 close to one.[75] Table 1 summarizes parameters and key moments used for the calibration.

Table 1. Parameterization

Parameter		Value	Target/Source
Household			
ρ	discount factor	0.05	standard value
g_L	pop. growth	0.02	standard value
h_{tot}	rel. skill share	0.15	Barro/Lee (2013)
h^*_{tot}	rel. skill share	0.05	see text
$\frac{L^*}{L}$	rel. size	0.66	see text
$\frac{w}{w^*}$	init. wage gap	4.0	see text
Production/Adoption			
α	capital share	0.4	standard value
δ_k	capital deprecication	0.05	standard value
δ_{X}	exit	0.04	IAB data
σ	substitution	2.5	Broda/Weinstein (2006)
β	static curvature	0.35	see text
θ	adv. backwardness	0.55	see text
f_{E}	entry cost	1.25	skill premium/waiting time
ν	adoption tech.	0.21	skill premium/waiting time
Innovation			
ϕ	dyn. externality	-1.0	see text
λ	congestion	0.9	see text
δ_{I}	exit innovation	0.04	IAB data
γ	research prod.	1	normalization
γ^*	research prod.	0.01	see text
ζ	initial idea share	0.99	initial output drop East

Note: The table provides the parameters used in the baseline calibration, which refers to the case of asymmetric integration.

Long-Run Effects. I first report the long-run effects of market integration before I turn attention to transition dynamics. Table 6 reports the results in the form of the cumulative effect, i.e., long-run level effect relative to a balanced growth path in autarky.

In the case of asymmetric integration where all innovation is produced in the West, wages of production workers in advanced economies fall by about 13% in real terms. This contrasts with wage gains for skilled labor of around 12%. Even though the technological frontier increases by

structure at time zero where the advanced economy's share of global patents equals ζ, with ζ being a function of b and the initial gap $\frac{A_{\text{F}}}{A^*_{\text{F}}}$.

[75]Note that the instantaneous change in GDP in the East at time zero equals approximately $\frac{\Delta Y^*}{Y^*} = -\frac{\sigma-1}{\sigma}\alpha(1-\alpha)$ for ζ close to one, which is the share of output paid to the owners of technology in autarky. For the baseline calibration this implies a drop of 14.4% over night.

7%, the adoption gap widens by 20% explaining weak wage growth for production labor in advanced economies. I also provide a measure of changes in GDP per capita y.[76] The negative effect of integration is still there, but somewhat weaker than the wage effects, consistent with GDP growth performing better than wage growth data over the past couple of decades, see for instance Autor et al. (2020). The role of intellectual property accumulation and asset income in the open economy explains this difference.

Figure 6. Long-Run Level Effects

Asymmetric Integration	West	East
χ	1.0	0.0
$\Delta \log w_L$	-0.129	1.276
$\Delta \log w_H$	0.123	1.448
$\Delta \log s$	0.252	0.172
$\Delta \log (y)$	-0.051	1.22
$\Delta \log z$	-0.196	1.209
$\Delta \log A_{\text{F}}^{\text{W}}$	0.067	0.067

Symmetric Integration	West	East
χ	0.602	0.398
$\Delta \log w_L$	0.253	1.639
$\Delta \log w_H$	0.253	1.584
$\Delta \log s$	0.0	-0.056
$\Delta \log (y)$	0.253	1.614
$\Delta \log z$	-0.0	1.386
$\Delta \log A_{\text{F}}^{\text{W}}$	0.253	0.253

Note. The table summarizes the long-run effect of market integration on income and inequality. The left panel uses the baseline calibration, while the right panel assume that skilled labor endowment and research productivity in the East coincide with the West. Note that the log change in the value of patents is infinite in the case of no innovation in the East in the integrated equilibrium. The adoption gap is defined relative to the global technological frontier $\log z_c = \frac{A_c}{A_{\text{F}}^{\text{W}}}$. For things to add up, I thus defined frontier technological growth as $\Delta \log A_{\text{F}}^{\text{W}} = \log A_{\text{F}}^{\text{W}} - \log A_{\text{F}}$ where A_{F} refers to the frontier in autarky, which resides in the West. Note that I net out exogenous long-run growth $\frac{\Delta \log L}{1-\phi}$ for non-stationary variables. The effects are smaller than in the job market paper version largely because the size of the foreign economy was assumed to be larger than in this version.

These negative findings contrast with the growth experience in the East: all workers gain massively albeit skilled labor gains relatively more. The gains are entirely accounted for by the adoption of frontier technology, which raises wages for production workers. Because adoption is a skill-intensive activity, there is pressure on the skill premium in emerging markets in spite of virtually zero research effort. Importantly, fast technology adoption in the East has a feedback effect on innovation in the West in the integrated equilibrium. This feedback effect induces increasing innovation at the cost of lower technology adoption, with overall negative long-run effects for the West as a whole.

To understand this long-run effects, note that in the baseline calibration the autarky equilibrium in the West is characterized by insufficient adoption. The share of skilled labor devoted to adoption

[76] Measuring GDP poses conceptual challenges. An issue arises as to where the value of ideas appears when computing GDP per capita, which itself might depend on firm's profit shifting. I propose the following measure, which corresponds to household consumption per capita. Note that with log utility, households will consume a fraction $\tilde{\rho}$ of their assets. I thus approximate real GDP per capita as $y := \frac{w_L L + w_H H + \tilde{\rho} B}{L+H}$ where B is the value of total household assets. This total value of assets does not change much because an increase in intellectual property roughly cancels with the relative fall in assets held in physical capital and production sector firms. However, because capital income is a substantial share of household income, including this term weakens the overall negative effect.

vis-a-vis innovation in the decentralized autarky equilibrium is roughly 40%, whilst a planner would allocate 2/3rds of the skilled work force to technology adoption.[77] This inefficient allocation not only depends on the learning externality in adoption, but also on the dynamic knowledge externality in innovation encoded in ϕ. If ideas are harder to find, shifting skilled labor into innovation has only modest effects on frontier growth, which are dominated by the negative impact of reduced technology adoption.[78]

Could it be, then, that all that is needed to reconcile the effects of the recent globalization wave on growth is a model where ideas are harder to find? Not quite. Using Jones (1995)'s model, which is a simplified version of the theory developed here,[79] the impact of integration on long-run wages are a log-linear function of the size of the population $L^{\frac{1}{1-\phi}}$. In the integrated equilibrium, specialization in innovation and production across countries delivers an increase in productivity and wages of the advanced economy by a factor of $(1+b^*)^{\frac{1}{1-\phi}}$, which is strictly positive. The assumption that ideas are harder to find is necessary but not sufficient condition, and need to be combined with a model where technology adoption complements frontier innovation. Only if there is too little adoption to begin with, and market integration with emerging market further amplifies this inefficiency, is there a chance for the model to generate sluggish growth in the aftermath of market integration. An implication of this is that subsidizing innovation in this model would be counterproductive for production workers in advanced economies: the skill premium would widen further, and more labor is reallocated away form domestic technology adoption, which was under-supplied to begin with.

In the case of symmetric integration, the gains from market integration are positive for all worker groups across all countries. Specifically, wages in advanced economies go up by 25%. This is, of course, just an instance of proposition 6: symmetric integration leaves the allocation of skill across sectors unchanged but delivers desirable scale effects as more skilled workers are engaged in idea production. The intuition is that while technology adoption abroad raises the incentive to innovation, foreign innovation reduces the incentive to innovate, and the two forces exactly cancel. The pro-growth effects of symmetric integration without adverse inequality effects paint a more realistic and complex picture of the link between growth and globalization. The model is consistent with the beneficial role of increasing integration across advanced economies in driving strong post-WW2 growth, while simultaneously allowing for the possibility of uneven and sluggish growth in the after-

[77] An important assumption underlying this welfare analysis hinges on the planner taking a national perspective. A planner that cares about world output instead faces a different trade-off. Intuitively, pushing out the frontier helps both the domestic and foreign economies, so more labor is devoted to producing frontier technology. Any welfare statement thus crucially depends on whether the scope of the analysis is global or national.

[78] I find that at a $\phi \approx 0$, all workers gain, and skill workers gain a lot resurrecting the strong pro-growth effect of integration. Again, the predictions of the theory are directly tied to how much harder ideas are to find.

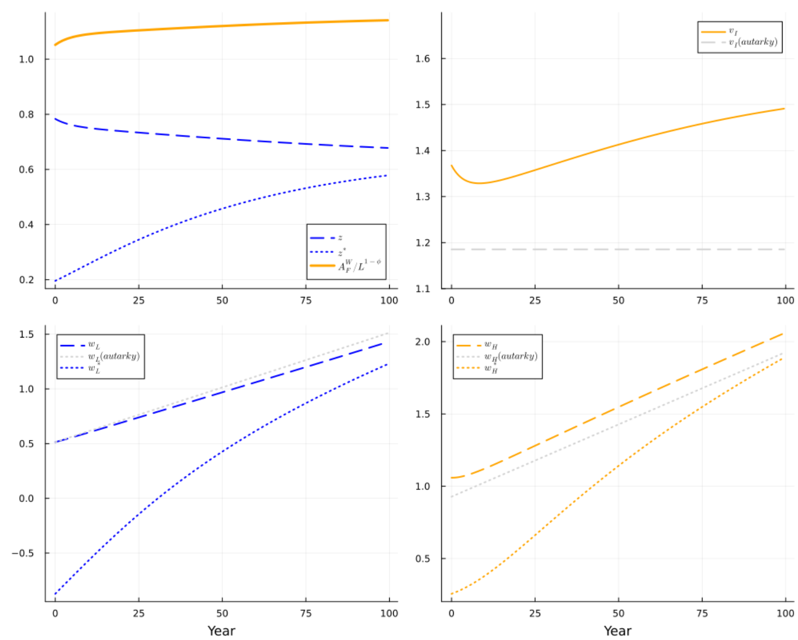
[79] If I drop the adoption margin, and make the production sector perfectly competitive by letting $f_E \to 0, \sigma \to \infty$, and I further assume $\lambda = 1$ and consider only one type of homogenous labor that can produce output or innovate, then the model coincides exactly with a version of Jones (1995) with log utility and $\lambda = 1$. Interestingly, the benevolent predictions of the benchmark model of Jones (1995) are virtually identical with the case of symmetric integration in a model with endogenous adoption.

math of market integration among asymmetric countries. The analysis so far has been restricted to long-run effects, which are slow-moving and mask important transitional dynamics, which I turn to next.

Transition dynamics. To simplify the transition dynamics, I make two assumptions. First, suppose households reinvest a constant fraction χ_{sav} of the final output good in each country where $\chi_{\text{sav}} = \left(\frac{\sigma-1}{\sigma}\alpha\right) \cdot \alpha \frac{g_L+g_{\text{F}}+\delta_k}{\rho+g_{\text{F}}+\delta_k}$ is set such that the saving rate is consistent with the long-run supply of savings from the household sector. This assumption is convenient as I don't have to solve the forward-looking household consumption-saving problem.[80] The second simplification is that I keep the normalized measure of firms in the production sector fixed at its long-run level m, which is the same in closed and open economy and across rich and poor countries.[81]

Figure 7 plots the transition dynamics in the case of asymmetric integration regarding wages and technology. Non-stationary variables are normalized by the long-run growth rate.

Figure 7. Wages & Technology along the Transition Path



Note. Based on baseline asymmetric calibration. Wages are plotted in logs, and the frontier level of technology is normalized by $L^{1-\phi}$ to net out exogenous long-run growth. Grey lines indicate the evolution of the variable in autarky from the point of view of advanced economies.

[80] In principal, one can solve a model with forward-looking household consumption, which would introduce consumption smoothing and inter-temporal trade. I prefer to simplify, which gives rise to a subtle issue: foreigners now hold domestic physical capital. Note that this has no impact on the long-run allocation but will allow for a temporarily higher interest rate in the emerging market along the transition path, which is a realistic feature of convergence growth.

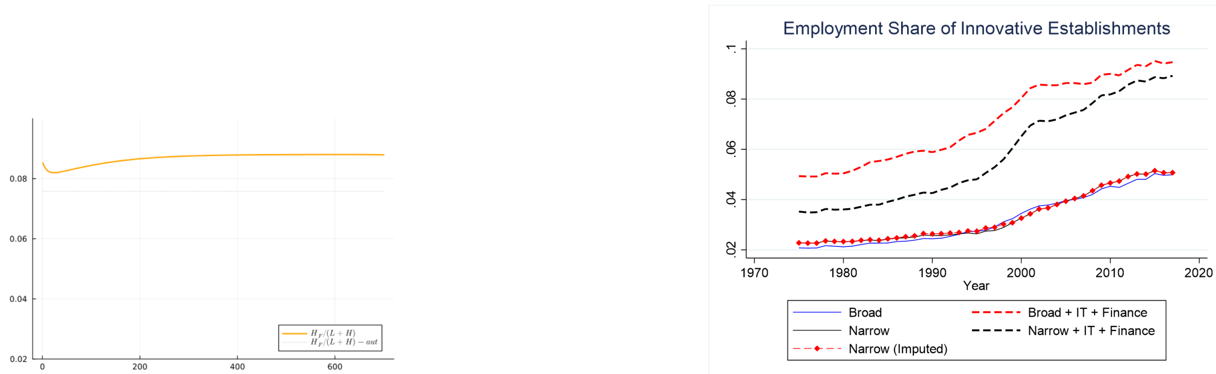
[81] The response of endogenous entry in the production sector is both computationally taxing, and matters little quantitatively. I plot the value of firms in the production sector, v, which does not fluctuate much suggesting that the channel is unimportant.

The left upper panel highlights that innovation responds quickly to a larger global market as the technological frontier expands quickly. The overall modest increase in the technological frontier is a consequence of ideas becoming harder to find. Technology adoption slows in advanced economies, leading to a widening adoption gap and a declining relative technology level z. In contrast, fast catch-up growth in emerging market is evident by the increase in the relative technology level z^*. The upper right panel plots a jump in the normalized value of an idea, v_{I}, driven by the initial impact of an increase in market size. This value displays non-monotone dynamics. The slight initial decline is explained by weak adoption in advanced economies, which is a drag on innovator profits. At the same time, the positive impact of the emerging market is growing over time as it travels closer to the technological frontier. Note that technology adoption matters twofold here. First, a rising the real wage makes the foreign market more attractive. Second, the waiting time τ^* is falling over time, further raising the net present discounted value of an innovation.

The lower two panels plot out the wage effects in each country. Production worker wages grow below trend for a long time in advanced economies as technology adoption is relatively slow-moving. In contrast, production workers in emerging markets experience exception wage growth driven by the advantage of backwardness. A sufficiently high skill endowment and a relatively large distance to the technological frontier allows for this exceptional growth spurt. Turning attention to high-skilled worker wages, they jump up in both countries. If one were to plot out the skill premium for each country, that skill premium would be falling over time for the emerging market as high returns to technology adoption when far away from the technological frontier drive up the skill premium beyond its long-run value along the transition path. The dynamics in the advanced economy are more subtle, and non-monotone. The skill premium initially spikes, then eases slightly due to sluggish adoption in the advanced economy, but eventually climbs again as technology adoption in the East continues to increase demand for skilled labor in the advanced economy's research sector. In this case of asymmetric integration, technology adoption in the East and rising returns to innovation in the West are two sides of the same coin.

A key statistic in the model is the relative share of labor devoted to research, defined as $\frac{H_{\mathrm{F}}}{H+L}$, which is plotted in figure 8. The left panel plots the share of employment in the research sector against total employment in the economy in the model, while the right panel plots the actual employment in research-intensive establishments broadly defined in Germany to gain a sense of whether the empirical magnitudes make sense. Details on the German micro data and more careful empirical analysis can be found in the Job Market Paper version of this paper, see Trouvain (2023). None of the empirical patterns are controversial, or unique to Germany, but I find it helpful to consider the German case as a case study since the country produces frontier technology and underwent a major globalization shock after the fall of the Iron Curtain. The model predicts an increase of research employment by roughly one percentage point, with the now familiar non-monotone pattern. If we contrast that with a broadly-defined notion of research employment, including headquarter services and finance, the magnitude is somewhat comparable to but still understates the large increase in research employment in Germany. While long-run structural change from manufacturing to services ought to matter for this finding, note that the research share really takes off in the mid 90s, in tandem with rising globalization and market integration with Eastern Europe.

Figure 8. Expanding Research Sector



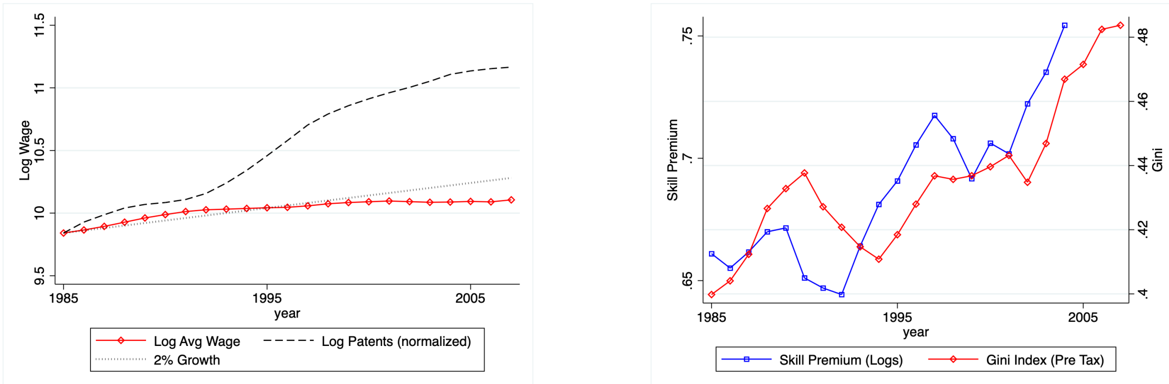
Note. Author's calculation based on BHP of the IAB. Narrow vs. Broad refers to 3 and 5 digit sectors, respectively. Additional details are in the appendix of Trouvain (2023).

A feature of the theory is its ability to reconcile rising innovative activity against the backdrop of stagnant real wages and weak TFP growth, as seen in figure 9. Wages grew at a rate above 2% up until 1995. From then onward, Germany experienced its worst two decades of economic growth since WW2, where per capita income growth fell to a meager 0.55% annually despite strong patent growth, a proxy for innovation, as can be seen in figure 9. Van Ark, O'Mahoney, and Timmer (2008) provide careful evidence showing that productivity growth slowed down dramatically. German TFP growth from 1995-2004 is estimated to be .3%, an all time low in post war history.[82]

The overall weak wage growth hides a great deal of heterogeneity across worker types with essentially zero growth for low-skilled workers, and robust growth for high skilled workers. Figure 9 shows the evolution of the skill premium, and the Gini Index, both of which shoot up in the mid 1990s, consistent with the model and the impact of market integration on the returns to innovation. This pattern of robust innovative activity, weak productivity growth, and a divergence in real wages across workers is not unique to the German economy but seems to hold across a number of advanced economies. This is a puzzle for benchmark models of endogenous growth, but the decoupling of innovation and wage growth visible in figure 9 is naturally accounted for by a model that acknowledges the importance of technology adoption, and considers the impact on globalization on the returns to "local adoption" vis-a-vis "global innovation". Similarly, rising profits and stock market valuations of

[82]See table 4 in Van Ark, O'Mahoney, and Timmer (2008).

Figure 9. Growth, Patents, and Inequality in Germany



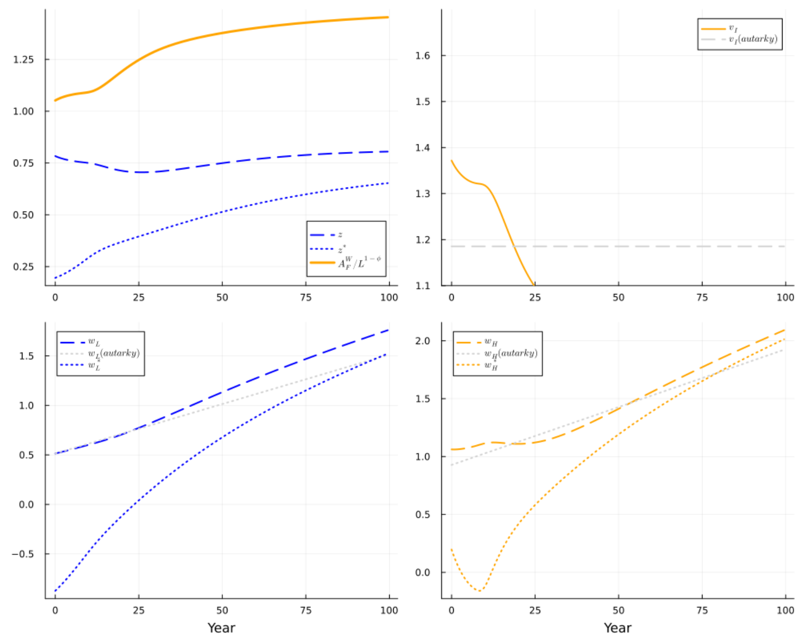
Data for patents comes from the Crios Patstata database, see Coffano and Tarasconi (2014). Wage data is computed based on the PWT version 09, combining real national gdp (not PPP) with their measure of the labor share and dividing thorough by the total population. Patents are normalized so that the wage level and patent level coincide in 1984. GDP per capita growth does better than wages, but still grows substantially below trend, leading to an overall growth slowdown. Data for the skill premium, denoted as $\log\left(\frac{w_H}{w}\right)$ where the wage rates are the price of one hour of skilled or production labor, comes from the KLEMS data version 07. Skill here refers to college-educated workers, group 3 in the Klems data. I do not make additional adjustments for efficiency units within skill group, which does not change the broad pattern. See the discussion and adjustments made in Buera et al. (2022) who also use the Klems data.The Gini index is pre tax and taken from the World Inequality Database of Alvaredo et al. (2020).

multinational companies are perfectly consistent with the increasing value of innovation in a globalization world.

This bleak scenario contrasts with the benevolent implication of symmetric market integration as I have argued before. I next plot the transition dynamics for the case when emerging markets converge in terms of fundamental research productivity and skill endowment to the level of advanced economies.

Interestingly, initially symmetric and asymmetric integration do not look very different initially: it takes time for the emerging market to improve their research productivity and skilled labor supply so in the early stages of convergence the emerging market largely adopts technology, which drives up the return to innovation and raises the skill premium, reducing technology adoption in the West. However, in the long-run it becomes apparent that the impact on production labor is fundamentally different. Innovation abroad pushes out the technological frontier, while simultaneously pushing skilled labor back into domestic technology adoption. This is good news for production workers, and even skilled workers are better off in this scenario as the fall in the skill premium is dominated by overall technological growth.

Figure 10. Wages & Technology along the Transition Path



Note. Based on symmetric integration where convergence in innovative capacity and skilled labor endowment takes roughly 30 years, see appendix. Wages are plotted in logs, and the frontier level of technology is normalized by $L^{1-\phi}$ to net out exogenous long-run growth. Grey lines indicate the evolution of the variable in autarky from the point of view of advanced economies.

Discussion & Empirical Evidence. The model is consistent with a number of empirical facts that have so far only considered in isolation: rising innovative effort and an increasing skill premium against the backdrop of sluggish productivity growth and overall wage stagnation in advanced economies. The key mechanism that accounts for the puzzling evolution of these secular trends is that innovation in a globalized world potentially comes at the cost of weakened local technology adoption, which is essential for wage growth of production workers in the framework. While a clean causal identification strategy for a theory of global growth is not available, I aim to make the theory more compelling in two ways. First, I address alternative explanations and will argue that it will be difficult to explain the growth patterns we have observed in the data. Second, I consider distinct microeconomic implications of the model that one could test using regional variation, and cite other empirical studies whose findings are consistent with the theory proposed here.

Alternative Hypotheses. Skill-biased technological change is perhaps the most influential explanations to account for the rise in the skill premium, and wage stagnation of production workers, see Katz and Murphy (1992) and Bound and Johnson (1992). While the evidence in favor of skill-biased technological change is strong, it is difficult to square it with overall wage stagnation because even though skill-biased technical change may favor of one worker group over another, it ultimately raises everyone's wage, see Acemoglu and Autor (2011) and Autor (2019) for a discussion of this point. A related literature has focused on the task-content of work and automation, which can generate more adverse effects of technological change on specific worker groups, see Autor, Levy, and Murnane (2003) and Acemoglu and Restrepo (2021). Even so, it is difficult to understand overall stagnant wages and slow productivity growth: while automation may reduce wages for some workers, it should raise overall GDP growth.[83] In contrast, in a model with endogenous skill-intensive adoption, rising inequality and weak productivity growth go hand in hand.

Similarly, a large literature has explored the link between globalization and inequality finding mixed results, see Helpman (2016) for an overview. Note that standard trade models predict gains from trade, and overall stagnation does not sit quite right with this implication. It may be, of course, that there strong pro-growth effects of market integration dominated by the some other forces like fading competition[84] or declining population growth to induce weak productivity growth. Interestingly, a slowdown in population growth through the lens of a model where ideas are harder to find (Jones, 1995) predicts a declining share of labor devoted to innovation. The fact that we seem to pour more and more resources into an activity that is getting harder and harder is puzzling. Taking the role of globalization serious resolves this tension as technology adoption abroad can sustain innovation in advanced economies even as ideas are getting harder to find.

Clearly, all of the aforementioned explanations must feature in any coherent account of growth around the turn of the 21st century. However, little attention has been paid to the role of technology adoption in a globalized world, which offers a simple explanation for a broad set of facts. I next turn to micro-econometric evidence in favor of the key mechanism.

Cross-Sectional Implications. The model has distinct implications for the uneven impact of globalization on innovation and technology adoption. The work of Andrews, Criscuolo, and Gal (2015) and Andrews, Criscuolo, and Gal (2016) seems largely in line with this interpretation as laggard firms, here firms in the production sector, seem to be loosing out. The main bottleneck to make the interpretation more compelling is that it is hard to come by comprehensive measures of technology adoption, which tends to be restricted to case studies of specific technologies.[85] I thus turn to regional variation within advanced economies, based on the logic that specialization in innovation vis-a-vis production is extremely uneven across space. If high-income regions have a persistent advantage in innovation, then market integration ought to further raise growth in innovative regions within advanced economies. This effect would be accompanied by a reallocation of skilled labor away from relatively

[83] See Aghion, Jones, and Jones (2017) for the powerful pro-growth effects of automation in endogenous growth model.

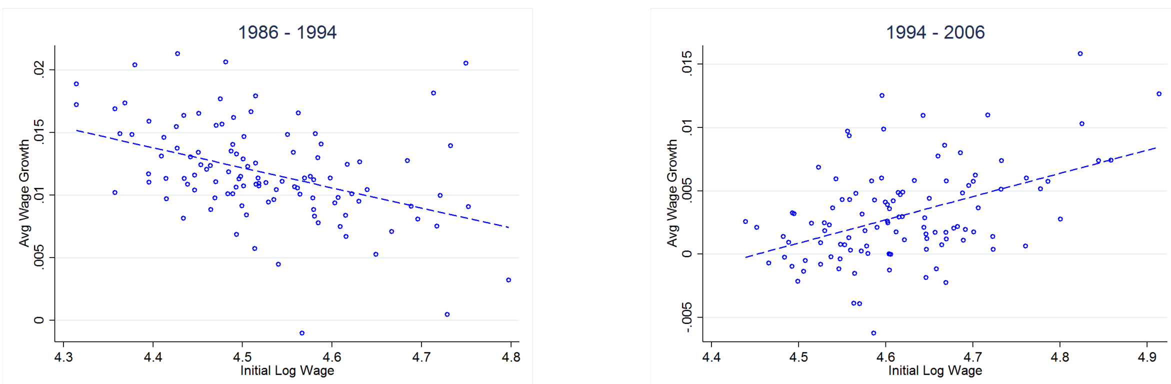
[84] The role of fading competition is hard to assess quantitatively as this strand of the literature has largely used models with strong scale effects where market size shocks have extreme predictions for long-run growth.

[85] See Griliches (1957)'s classic study of the diffusion of hybrid seeds in the US, Comin and Hobijn (2010b) combine a number of technologies and study their diffusion across countries over time. Anzoategui et al. (2019) use survey data on the diffusion of specific technologies in the US and the UK. The latter study is consistent with the important role of technology adoption in accounting for the growth slowdown although their focus is ultimately transitory and driven by cyclical variation, in contrast to my framework where a globalization-induced increase in the skill premium creates persistent level effects.

poorer regions and coincide with weak growth in production-focused regions as technology stalls. I make this argument more carefully in the working paper version (Trouvain, 2023), and focus on the simple cross-regional growth patterns for Germany in this version of the paper.

Figure 11 plots average wage growth, defined as the total wage bill of full-time employees over total full-time employment, against the log of the initial average real wage for a local labor market in West Germany,[86] following Baumol (1986). While wage growth in the early period from 1986 – 1994 was, on average, higher for laggard regions. These growth patterns are turned upside down in the 2000s, where high-income places grew relatively fast while laggard regions stagnated.[87] To the extent that laggard regions are more focused on production, and frontier regions host most of the innovation, the changing growth patterns are consistent with rising returns to innovation in the aftermath of global market integration.

Figure 11. Regional Convergence in Germany



Using data from the BHP establishment sample, the figure plots average wage growth against initial the initial average wage in real terms. The plot shows how growth pre 1994 was biased towards lagging regions, while from 1994 onwards growth was biased towards high income regions. I stop short of the financial crisis, but have looked at convergence patterns from 206 - 2015 as well which are mostly neutral with a regression coefficient statistically indistinguishable from zero at standard levels of significance. See the appendix for plots for high, middle, and low skilled wages separately. A common concern is that international trade, and in particular import exposure following Autor, Dorn, and Hanson (2013), fully explains weak growth in laggard regions. To consider the effect of import exposure on wage growth, I run a convergence regression with an additional import exposure variable as control. Import competition accounts for virtually none of the stagnation in laggard regions.

These changing regional convergence patterns are more broadly true across advanced economies including the USA (Berry and Glaeser, 2005; Ganong and Shoag, 2017; Giannone, 2017; Rubinton, 2020), and thus should be uncontroversial. The important difference to this spatially-focused literature is that their explanations are based on skill-biased technological change, which, as argued before,

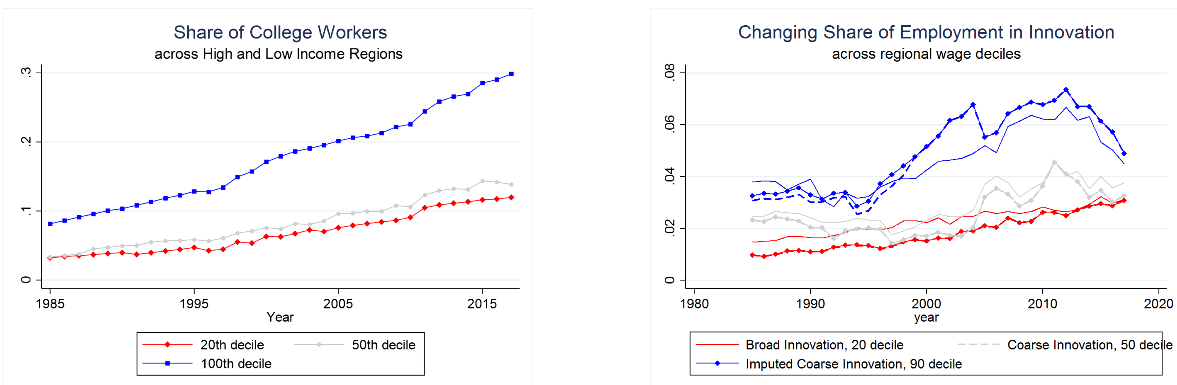
[86] Clearly, German integration poses econometric challenges. However, note that internal integration, at least through the lens of the model, does not generate a bias towards innovation as skilled labor would earn a high return in Eastern Germany to help adopt technology.

[87] It is likely that fast growth in high income places is still an understatement due to top-coding issues in the German data. The IAB provides average wages on the establishment level that use the imputation procedure in Card, Heining, and Kline (2013) to deal with the fact that as much as 10% of wage observations are top coded. This procedure relies on a log normal model of the wage distribution which is conservative considered against the thick right tail of the income distribution.

induces relatively fast GDP growth. In contrast, in a model with endogenous technology adoption, a meaningful tradeoff emerges. For my baseline calibration, frontier growth does not compensate for weak technology adoption in the hinterlands, consistent with the aggregate growth slowdown and figure 11 when paying attention to the scale of the y-axis: the growth drag from laggard regions is too large and dominates.

Weak technology adoption in the cross-section of regions is ultimately driven by a reallocation of skilled labor, which is born out in the data. Figure 12 displays the uneven evolution of the skilled labor share across regions in Germany. The general equilibrium structure of the model makes clear that the acceleration in skill growth in innovative centers comes at the cost of production-focused regions, which could appropriately be described as brain drain.

Figure 12. Share of College Workers across Regions



These plots compute skill share and employment in innovation across high and low income regions in Germany by grouping regions into wage deciles and computing simple averages. The plots are purely cross-sectional in the sense that I assign labor markets into bins each year so that for example the set of places in the top bin can change every year. In practice, whether one fixed the income ranking in 1994 instead does not change the broad patterns. There is substantial sampling variation within each region, however, and the cross sectional plots is smoother, which is why I prefer it.

The final piece of evidence in favor of the role of skilled labor in technology adoption, which is at the heart of the mechanism, comes from seemingly unrelated studies. Using micro data and a causal estimation design based on cross-regional variation, Lewis (2011), Beaudry, Doms, and Lewis (2010), and Imbert et al. (2022), provide compelling evidence that a change in the local skill mix towards less skilled workers reduces a local labor market's adoption of new technology. Similarly, classic arguments related to growth at the eve of the industrial revolution highlight the role of skilled labor and labor scarcity as drivers of growth, see Mokyr (2009) and Allen (2009), respectively. Recent work of Voth, Caprettini, and Trew (2023) provides evidence in favor of both views. The theory I have proposed is consistent with these accounts: skill is key for adoption, and shocks that push down the skill premium, be it because of a skilled labor supply shock, or a negative shock to production labor, raise the incentives to adopt technology.

5 Conclusion

Global market integration across advanced economies and emerging markets changes the returns to innovation vis-a-vis technology adoption. The research sector in advanced economies expands, while domestic technology adoption stalls. I make this argument precise by generalizing the model of Jones (1995) to include an endogenous technology adoption gap. The theory highlights how innovation and technology adoption are complementary on the market for ideas, but at the same time compete for skilled labor on factor markets. This leads to a novel role for the skill premium, which directly impacts productivity through its effect on equilibrium adoption effort.

In my calibration, weak domestic technology adoption entirely erases gains from additional innovation in the aftermath of market integration between advanced economies and emerging markets. The mechanism can generate sizable real wage losses for production workers in rich countries, and explains weak aggregate growth in advanced economies despite rising innovative efforts. In spite of this dire prediction, openness and globalization can play a powerful role in sustaining long-run growth due to the inherent non-rivalry of ideas when emerging markets converge all the way and start to contribute to the technological frontier. Concerns about the adverse effects of the ability of emerging markets to compete with advanced economies in the research sector are misplaced through the lens of the model: innovation in emerging markets would push out the technological frontier, and simultaneously induce a reallocation of skilled labor toward adoption activity within advanced economies generating broad-based wage growth for all workers.

Much work remains to be done to discipline the innovation-adoption tradeoff that is the focus of the paper, not least for a lack of comprehensive measures of technology adoption. Yet, I hope that the framework's simplicity and ability to explain several patterns in the data all at once will contribute toward a better understanding of the nexus of technological change, inequality, and globalization.

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A Theory Appendix

A.1 Production Firm

A.1.1 Static minimization problem of firm in production sector

Optimality can be split into a number of steps, where first I begin by deriving the efficient demand for each capital good, x_z, holding A fixed. Without loss of generality, one can think of the capital goods x_j as contained in the interval $[0, A]$ where $\int_0^A dj = A$. Given total expenditure on capital goods $\int p_j x_j dj = p_j x$ where $\int x_j dj = x$, I can ask how much expenditure is spend on each particular variety. The problem reads

$$\begin{array}{ll} \max & \int_0^A \left(\frac{x_j}{\alpha}\right)^\alpha dj \\ s.t. & \int p_j x_j dj \leq I. \end{array} \tag{50}$$

This well-known problem (Dixit and Stiglitz, 1977) leads to the following first order condition

$$\frac{x_j}{x_z} = \left(\frac{p_j}{p_z}\right)^{-\frac{1}{1-\alpha}},$$

and since the capital goods are homogeneous it follows that $x_j = x_k \, \forall j, k$. As a consequence, the total quantity of each individual capital good variety must read $p_j x_j = \frac{p_x \tilde{x}}{A}$ where the last equality holds because of the symmetry assumption.

Now I can substitute this into the firm production function and find the minimal cost of producing one unity of output, given factor prices. This leads to the following cost minimization problem

$$\begin{array}{ll} \min & wl + p_x \tilde{x} \\ s.t. & \left(\int_0^A \left(\frac{\tilde{x}}{\alpha}\frac{1}{A}\right)^\alpha dj\right)\left(\frac{l}{1-\alpha}\right)^{1-\alpha} \geq 1 \end{array}$$

The problem further simplifies to

$$\begin{array}{ll} \min & wl + p_x \tilde{x} \\ s.t. & \left(\frac{\tilde{x}}{\alpha}\right)^\alpha \left(\frac{Al}{1-\alpha}\right)^{1-\alpha} \geq 1 \end{array}$$

which has the convenient Cobb-Douglas structure with labor-augmenting technological change. The first order conditions lead to the constant ratio of expenditure shares on labor and capital

$$\frac{p_x \tilde{x}}{wl} = \frac{\alpha}{1-\alpha}$$

Together with the binding constraint, $\left(\frac{\tilde{x}}{\alpha}\right)^\alpha \left(\frac{Al}{1-\alpha}\right)^{1-\alpha} = 1$, the cost-minimizing bundle of labor and capital leads to a marginal (and average) unit cost of

$$mc \quad = \quad (p_x)^{\alpha} \left(\tfrac{w}{A}\right)^{1-\alpha} \quad .$$

Average and marginal cost coincide since the production function features constant returns in capital and labor, conditional on A.

This constant-marginal cost results is important as it simplifies the firm's price setting problem, taking aggregate variables as given. Formally, the problem reads

$$\max_p \quad Yp^{-\sigma}\left[p - mc\right]$$

which leads to the well-known constant markup over marginal cost,

$$p = \frac{\sigma}{\sigma - 1} mc.$$

This constitutes a solution to the static firm problem. Since profits are strictly decreasing in marginal cost, it is indeed optimal to achieve lowest cost and then charge a constant markup over marginal cost.

A.1.2 Dynamic Firm Problem and adoption Gap

The firms' adoption problem is summarized in the following HJB equation

$$\begin{aligned} V\left(r_t + \delta_{\mathrm{X}}\right) &= \max_{h_{i,t}} \pi^0\left(A_{i,t}\right) - w_{H,t}h_{i,t} + \dot{A}_{i,t}\partial_A V + \dot{V} \\ \text{s.t.} & \\ \dot{A}_{i,t} &= \nu A_{\mathrm{F},t}^{1-\theta} A_{i,t}^{\theta} h_{i,t}^{\beta} - \delta_{\mathrm{I}} A_{i,t}, \end{aligned} \tag{51}$$

where δ_{X} is an exogenous firm death shock and $\partial_A V := \frac{\partial V}{\partial A}$ denotes the partial derivative

To solve the production firm's adoption problem, it is useful to rewrite the problem using a normalized value function $v = \frac{V}{w_L}$, as well as normalizing the state variable A by A_F, i.e. the state becomes z. With these assumptions, I obtain a system that is stationary in the steady state. In the log utility case with $r = \rho + g_{\mathrm{F}}$ along a balanced growth path, this leads to the following recursive formulation of the firm adoption problem,

$$\begin{aligned} v\left(\rho + \delta_{\mathrm{X}}\right) &= \max_h \frac{\pi_t^o(z)}{w_L} - sh + \dot{z}\partial_z v + \dot{v} \\ s.t. & \\ \dot{z} &= \nu z^{\theta} h^{\beta} - \left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) z. \end{aligned} \tag{52}$$

A solution to the program (52) needs to satisfy the following first order condition

$$\left\{\frac{\beta\nu z^{\theta}\partial_z v}{s}\right\}^{\frac{1}{1-\beta}} = h \,. \tag{53}$$

Equation (53) captures the tradeoff of the effect on firm value of a marginal increase in h relative to its cost s. In anticipation of the solution, I derive the derivative of h with respect to z and t, which yields

$$\begin{aligned}\frac{\partial_{zz}v}{\partial_z v}+\frac{\theta}{z} &= (1-\beta)\,\frac{\partial_z h}{h}\\ \frac{\partial_z \dot{v}}{\partial_z v}-\frac{\dot{s}}{s} &= (1-\beta)\,\frac{\partial_t h}{h}.\end{aligned}$$

Next, derive the envelope condition of the HJB equation to get

$$\begin{aligned}(\partial_z v)\left(\rho+\delta_{\mathrm{X}}\right) &= \frac{\partial_z\pi}{w_L}+\dot{z}\left(\partial_{zz}v\right)+\left(\theta z^{\theta-1}\nu h^{\beta}-\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)\right)(\partial_z v)+\partial_z\dot{v}\\ (\partial_z v)\left(\rho+\delta_{\mathrm{X}}+(1-\theta)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)\right) &= \frac{\partial_z\pi}{w_L}+\dot{z}\left(\partial_{zz}v\right)+(\partial_z v)\,\frac{\theta}{z}\left(z^{\theta}\nu h^{\beta}-z\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)\right)+(\partial_z v)\left\{(1-\beta)\,\frac{\partial_t h}{h}+\frac{\dot{s}}{s}\right\}\\ (\partial_z v)\left(\rho+\delta_{\mathrm{X}}+(1-\theta)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)\right) &= \frac{\partial_z\pi}{w_L}+(\partial_z v)\left(\frac{\partial_{zz}v}{\partial_z v}+\frac{\theta}{z}\right)\dot{z}+(\partial_z v)\left\{(1-\beta)\,\frac{\partial_t h}{h}+\frac{\dot{s}}{s}\right\}\\ \rho+\delta_{\mathrm{X}}+(1-\theta)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right) &= \frac{\partial_z\pi}{w_L}\frac{1}{\partial_z v}+(1-\beta)\,\frac{\dot{h}}{h}+\frac{\dot{s}}{s}.\end{aligned}$$

Now I can substitute in the first order condition and use the fact that I know the derivative of the profit function to get

$$\begin{aligned}\frac{\dot{h}}{h} &= \frac{1}{1-\beta}\left\{\left(\rho+\delta_{\mathrm{X}}+(1-\theta)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)\right)-\frac{1}{\partial_z v}\left[\frac{\pi^{o}}{w_L}\frac{(1-\alpha)(\sigma-1)}{z}\right]-\frac{\dot{s}}{s}\right\}\\ \frac{\dot{h}}{h} &= \frac{1}{1-\beta}\left\{\left(\rho+\delta_{\mathrm{X}}+(1-\theta)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)\right)-\frac{\beta\nu z^{\theta}h^{\beta-1}}{s}\left[\frac{\pi^{o}}{w_L}\frac{(1-\alpha)(\sigma-1)}{z}\right]-\frac{\dot{s}}{s}\right\}\end{aligned}$$

Moreover, recall that the law of motion of relative technology reads

$$\frac{\dot{z}}{z} = \nu z^{\theta-1}h^{\beta}-\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right).$$

In the steady state, we have that

$$h^{1-\beta} = \frac{1}{s}\frac{\beta(1-\alpha)(\sigma-1)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\rho+\delta_{\mathrm{X}}+(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}\left[\frac{\pi^{o}}{w_L}\right]\frac{\nu z^{\theta-1}}{(g_{\mathrm{F}}+\delta_{\mathrm{I}})} \tag{54}$$

$$z^{1-\theta} = \frac{\nu h^{\beta}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}} \tag{55}$$

If we combine these two equations one can see that a constant spending on learning activity follows

$$hs = \frac{\beta(1-\alpha)(\sigma-1)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\rho+\delta_{\mathrm{X}}+(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}\left[\frac{\pi^{o}}{w_L}\right].$$

This leads to an inequality that needs to be satisfied for the equilibrium to be well-defined, namely

$$\beta\left(1-\alpha\right)\left(\sigma-1\right) \quad < \quad \tfrac{\rho+\delta_{\mathrm{X}}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}+\left(1-\theta\right).$$

The left hand side represents the additional benefit of improving your productivity, which combines the diminishing returns in learning (β) with the elasticity of the profit function ($\left(\sigma-1\right)\left(1-\alpha\right)$). The right hand side consist of effective costs in steady state, which is related to effective discounting as well as the advantage of backwardness. The firm needs to take into account that as it climbs up the technological ladder, the pull force introduced through the advantage of backwardness diminishes. This gives rise to an endogenous adoption gap as a function of the relative price of skill. Moreover, climbing up the ladder is costly when discounting is high since the benefits only accrue in the future.

A.1.3 Firm value function off and on the balanced growth path

Suppose that free entry into innovation and production holds. In that case, it must be that $f_{\mathrm{E}} = v\left(t,z\right)$. Now the value function solves the HJB

$$\left(r+\delta_{\mathrm{X}}-g_{w_L}\right)v \quad = \quad \max_h \tfrac{\pi^o}{w_L}-sh+\dot{z}\left(\partial_z v\right)+\dot{v}.$$

This dynamic HJB equation is tied to the free entry condition in a useful way, see the discussion in Peters and Walsh (2019), which I leverage in the next steps. Note that totally differentiating $f_{\mathrm{E}} = v\left(z,t\right)$ with respect to time t implies $\dot{z}\left(\partial_z v\right) = -\dot{v}$. I use this relationship to simplify the HJB equation where it must be understood that h solves the dynamic adoption problem. Rearranging yields

$$v \quad = \quad \frac{\frac{\pi^o}{w_L}-sh}{r_t+\delta_{\mathrm{X}}-g_{w_L}}$$

where I did not assume anything about the stationarity of any of the variables.

Care must be taken for the case when the free entry condition does not hold. In that case, I can compute the firm value by piecing together the part of the problem where no entry occurs (so I know exactly what the measure of firms is and hence can back out profits and the optimal adoption decision) plus the value when free entry is again binding. This is relevant because entry is going to be responsive to learning activity, which pushes down current profits and might thus command a smaller measure of firms in equilibrium.

A.2 Research Sector

Long-run growth rate. To see why the long-run growth rate equals $\frac{g_L}{1-\phi}$, rearrange the resource constraint in innovation

$$
\begin{aligned}
\dot{A}_{\mathrm{F}} &= \gamma A_{\mathrm{F}}^{\phi}\left(\frac{H_{\mathrm{F}}}{L}\right)^{\lambda-1} H_{\mathrm{F}} - \delta_{\mathrm{I}} A_{\mathrm{F}} \Rightarrow \\
(g_{\mathrm{F}} + \delta_{\mathrm{I}}) &= \gamma A_{\mathrm{F}}^{\phi-1} h_{\mathrm{F}}^{\lambda-1} H_{\mathrm{F}} \\
(g_{\mathrm{F}} + \delta_{\mathrm{I}}) &= \gamma \frac{L}{A_{\mathrm{F}}^{1-\phi}} h_{\mathrm{F}}^{\lambda-1} \frac{H_{\mathrm{F}}}{L} \Rightarrow \\
a_{\mathrm{F}} &= \frac{\gamma h_{\mathrm{F}}^{\lambda}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}
\end{aligned}
$$

where I used the definitions $a_{\mathrm{F}} = \frac{A_{\mathrm{F}}^{1-\phi}}{L}$ and $h_{\mathrm{F}} = \frac{H_{\mathrm{F}}}{L}$. It is easy to see now that along a balanced growth path with positive population growth and $\phi < 1$, it must be that $g_{\mathrm{F}} = \frac{g_L}{1-\phi}$.[88]

Similarly, the law of motion of normalized ideas follows from noting that by definition

$$
\frac{\dot{a}_{\mathrm{F}}}{a_{\mathrm{F}}} = (1-\phi)\, g_{\mathrm{F}} - g_L
$$

and after substituting out $g_{\mathrm{F}} = \gamma A_{\mathrm{F}}^{\phi-1}\left(\frac{H_{\mathrm{F}}}{L}\right)^{\lambda-1} H_{\mathrm{F}} - \delta_{\mathrm{I}}$,

$$
\begin{aligned}
\frac{\dot{a}_{\mathrm{F}}}{a_{\mathrm{F}}} &= (1-\phi)\left[\gamma A_{\mathrm{F}}^{\phi-1}\left(\frac{H_{\mathrm{F}}}{L}\right)^{\lambda-1} \frac{H_{\mathrm{F}}}{L} L - \delta_{\mathrm{I}}\right] - g_L \\
&= (1-\phi)\left[\frac{\gamma h_{\mathrm{F}}^{\lambda}}{a_{\mathrm{F}}} - \delta_{\mathrm{I}}\right] - g_L \\
&= (1-\phi)\left[\frac{\gamma h_{\mathrm{F}}^{\lambda}}{a_{\mathrm{F}}} - \left(\delta_{\mathrm{I}} + \frac{g_L}{1-\phi}\right)\right]
\end{aligned}
$$

which implies

$$
\dot{a}_{\mathrm{F}} = (1-\phi)\left[\gamma h_{\mathrm{F}}^{\lambda} - a_{\mathrm{F}}\left(\delta_{\mathrm{I}} + \frac{g_L}{1-\phi}\right)\right]
$$

as in the main text.

[88] Note that if I had chosen the congestion force using $f_{\mathrm{R}} = H_{\mathrm{F}}^{1-\lambda}$, the link between aggregate per capita growth and population growth would be $g_{\mathrm{F}} = \frac{\lambda}{1-\phi} g_L$. In an earlier version, I used this exact formulation, which works well in the closed economy. In the open economy, small countries would have a large incentive to specialize in innovation to the extent that this congestion externality is "local" as opposed to global.

Decentralized equilibrium. Assume free entry holds

$$V_{\mathrm{I}} \quad = \quad \frac{f_{\mathrm{R}} w_H}{A_{\mathrm{F}}^{\phi}} \quad . \tag{56}$$

Totally differentiating (56) with respect to time t yields

$$\dot{V}_{\mathrm{I}} = V_{\mathrm{I}} \left((1-\lambda) \left(g_{H_{\mathrm{F}}} - g_L \right) + g_w + g_s - \phi g_{\mathrm{F}} \right) \tag{57}$$

where I used the fact that $f_{\mathrm{R}} = \frac{\left(\frac{H_{\mathrm{F}}}{L}\right)^{1-\lambda}}{\gamma}$.

Recall the expression for the net present value of an idea

$$V_{\mathrm{I}} \quad = \quad \int_{t+\tau}^{\infty} \exp\left(- \int_t^u \left(r_v + \delta_{\mathrm{I}} \right) dv \right) \pi_{\mathrm{I},u} du \quad . \tag{58}$$

Totally differentiating (58) with respect to time yields

$$\dot{V}_{\mathrm{I}} = -\exp\left(- \int_t^{t+\tau} \left(r_v + \delta_{\mathrm{I}} \right) dv \right) \pi_{\mathrm{I},t+\tau} \cdot \left[1 + \dot{\tau}_t \right] + \left(r_t + \delta_{\mathrm{I}} \right) V_{\mathrm{I}}, \tag{59}$$

which expresses the value of an innovation in terms of its properly discounted flow profits, $\exp\left(- \int_t^{t+\tau} \left(r_v + \delta_I \right) dv \right) \pi_{I,t+\tau} \cdot$ $\left[1 + \dot{\tau}_t\right]$, taking into account changes in the waiting time $\dot{\tau}$, as well as appreciation $\dot{V}_{\mathrm{I}}$. Combining (57) and (59) yields

$$V_{\mathrm{I}} = \frac{\exp\left(- \int_t^{t+\tau} \left(r_v + \delta_{\mathrm{I}} \right) dv \right) \pi_{\mathrm{I},t+\tau} \cdot \left[1 + \dot{\tau}_t \right]}{r_t - g_{w_L} - g_s + \delta_{\mathrm{I}} - (1-\lambda) \left(g_{H_{\mathrm{F}}} - g_L \right) + \phi g_{\mathrm{F}}}$$

which is identical to the expression in the paper.

In anticipation of solving for a balanced growth path equilibrium, I normalize the value function using $\frac{w_L}{A_{\mathrm{F}}^{\phi}}$ as normalizing factor so that $v_{\mathrm{I}} = \frac{V_{\mathrm{I}}}{w_L} A_{\mathrm{F}}^{\phi}$, and by free entry $v_{\mathrm{I}} = \frac{s}{\gamma} h_{\mathrm{F}}^{1-\lambda}$. Using this normalization

$$\begin{aligned}
v_{\mathrm{I}} &= \frac{A_{\mathrm{F},t}^{\phi}}{w_{L,t}} \frac{\exp\left(- \int_t^{t+\tau} \left(r_v + \delta_{\mathrm{I}} \right) dv \right)}{r_t - g_{w_L} - g_s + \delta_{\mathrm{I}} - (1-\lambda) \left(g_{H_{\mathrm{F}}} - g_L \right) + \phi g_{\mathrm{F}}} \pi_{\mathrm{I},t+\tau} \left(1 + \dot{\tau}_t \right) \\
v_{\mathrm{I}} &= \frac{A_{\mathrm{F},t}^{\phi}}{w_{L,t}} \frac{\exp\left(- \int_t^{t+\tau} \left(r_v + \delta_{\mathrm{I}} \right) dv \right)}{r_t - g_{w_L} - g_s + \delta_{\mathrm{I}} - (1-\lambda) \left(g_{H_{\mathrm{F}}} - g_L \right) + \phi g_{\mathrm{F}}} \frac{\alpha w_{L,t+\tau} L_{\mathrm{P},t+\tau}}{A_{\mathrm{F},t+\tau} z_{t+\tau}} \left(1 + \dot{\tau}_t \right) \\
v_{\mathrm{I}} &= \frac{\exp\left(- \int_t^{t+\tau} \left(r_v + \delta_{\mathrm{I}} - g_{w_L} + \phi g_{\mathrm{F}} \right) dv \right)}{r_t - g_{w_L} - g_s + \delta_{\mathrm{I}} - (1-\lambda) \left(g_{H_{\mathrm{F}}} - g_L \right) + \phi g_{\mathrm{F}}} \frac{\alpha L_{\mathrm{P},t+\tau}}{A_{\mathrm{F},t+\tau}^{1-\phi} z_{t+\tau}} \left(1 + \dot{\tau}_t \right) \\
v_{\mathrm{I}} &= \frac{\exp\left(- \int_t^{t+\tau} \left(r_v + \delta_{\mathrm{I}} - g_{w_L} + \phi g_{\mathrm{F}} \right) dv \right)}{r_t - g_{w_L} - g_s + \delta_{\mathrm{I}} - (1-\lambda) \left(g_{H_{\mathrm{F}}} - g_L \right) + \phi g_{\mathrm{F}}} \frac{\alpha l_{\mathrm{P},t+\tau}}{a_{\mathrm{F},t+\tau} z_{t+\tau}} \left(1 + \dot{\tau}_t \right)
\end{aligned}$$

where $h_{\mathrm{F}} := \frac{H_{\mathrm{F}}}{L}$ and $a_{\mathrm{F}} = \frac{A_{\mathrm{F}}^{1-\phi}}{L}$ are normalized variables that are constant along a balanced growth path.

Given log utility, the real rate equals $r = \rho + g_{\mathrm{F}}$. Along a balanced growth path, v_I simplifies to

$$\begin{aligned} v_{\mathrm{I}} &= \frac{\exp\left(-\int_t^{t+\tau}\left(\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) dv\right)}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}} z} \\ &= \frac{\exp\left(\frac{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} \log z\right)}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}} z} \\ &= \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}}}{a_{\mathrm{F}}} \end{aligned}$$

where I used $\dot{s} = \dot{\tau} = 0$, and $g_{W_L} = g_A = g_{\mathrm{F}} = \frac{g_L}{1-\phi}$, and the definition $\tilde{\rho} = \rho - g_L$.

Using free entry and in particular $\frac{s}{\gamma} h_{\mathrm{F}}^{1-\lambda} = v_{\mathrm{I}}$ I can derive normalized equilibrium demand for skilled labor

$$h_{\mathrm{F}} = \left\{ \frac{\gamma}{s} \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}}}{a_{\mathrm{F}}} \right\}^{\frac{1}{1-\lambda}}.$$

Combining this with the resource constraint for ideas in the steady state, $a_{\mathrm{F}} = \frac{\gamma h_{\mathrm{F}}^{\lambda}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}$, leads to

$$h_{\mathrm{F}} = \left\{ \frac{1}{s} \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}} \right\},$$

which is the same expression as in the main part of the paper.

A.2.1 Waiting time for innovator

The waiting time τ for an innovator's idea to be adopted can be derived as follows. Recall equation (18). Use an integrating factor and note that on the balanced growth path with a constant adoption gap, $g_A = g_F$. Normalizing the time of entry to zero ($t_{\mathrm{E}} = 0$) so that calendar time t coincides with waiting time, I can derive the waiting time as a solution to the following differential equation

$$\dot{W}_t = -\delta_{\mathrm{I}} W_t - A_t \left(g_A + \delta_{\mathrm{I}}\right).$$

Using an integrating factor $e^{\delta_{\mathrm{I}}t}$

$$\begin{aligned}
\int_0^t \frac{\partial \exp(\delta_{\mathrm{I}}u)W_u}{\partial u} &= -\int_0^t \exp(\delta_{\mathrm{I}}t)\, A_u \left(\delta_{\mathrm{I}} + g_A\right) du \\
\exp(\delta_{\mathrm{I}}t)\, W_t - W_0 &= -A_0 \int_0^t e^{\int_0^u g_A(x)+\delta_{\mathrm{I}}dx} \left(\delta_{\mathrm{I}} + g_A\right) du \\
\exp(\delta_{\mathrm{I}}t)\, W_t - W_0 &= -A_0 \int_0^t e^{\int_0^u \zeta_x dx} \left(\zeta_u\right) du \\
\exp(\delta_{\mathrm{I}}t)\, W_t - W_0 &= -A_0 \left(e^{\int_0^t g_A(x)+\delta_{\mathrm{I}}dx} - 1\right) \\
W_t - W_0 \exp(-\delta_{\mathrm{I}}t) &= A_0 \exp(-\delta_{\mathrm{I}}t) - A_t \\
W_t - (A_{\mathrm{F},0} - A_0)\exp(-\delta_{\mathrm{I}}t) &= A_0 \exp(-\delta_{\mathrm{I}}t) - A_t \\
W_t - (A_{\mathrm{F},0})\exp(-\delta_{\mathrm{I}}t) &= -A_t \\
(A_{\mathrm{F},0})\exp(-\delta_{\mathrm{I}}t) - A_0 e^{\int_0^t g_A(u)du} &= W_t \\
(A_{\mathrm{F},0})\exp(-\delta_{\mathrm{I}}t)\left(1 - z_0 e^{\int_0^t g_A(u)+\delta_{\mathrm{I}}du}\right) &= W_t.
\end{aligned}$$

Now set $W(0,t) = 0$, which is the point in time when the waiting time is zero. From the previous derivations it is clear that this happens when

$$1 = z_0 e^{\int_0^t g_A(u)+\delta_{\mathrm{I}}du}, \tag{60}$$

i.e., the waiting time is implicitly defined by (60). Taking logs and rearranging yields

$$-\frac{\log z_0}{\frac{\int_0^t g_A(u)+\delta_{\mathrm{I}}du}{t}} = t.$$

In general, instead of starting at time zero, entering cohorts start in calendar time t and hit the market at $t + \tau_t$, so the expression generalizes to

$$\tau = -\frac{\log z_t}{\frac{\int_t^{t+\tau} g_A(u)+\delta_{\mathrm{I}}du}{\tau}}, \tag{61}$$

in line with the claim in the main text.

This waiting time is an endogenous object that depends on inventors' and adopters' choices, not just in t but also in periods going forward. In the steady state, however, the expression collapses to a simple statistic

$$\tau = -\frac{\log z}{g_A + \delta_{\mathrm{I}}}.$$

Next, I derive the time derivative $\dot{\tau}$ which is important to compute transition dynamics. Note that (61) implies

$$\tau\delta_{\mathrm{I}} \quad = \log A_{\mathrm{F},t} - \log A_{t+\tau}$$

I totally differentiate this expression to obtain

$$\begin{aligned} d\tau\delta_{\mathrm{I}} &= g_{\mathrm{F}}(t)\,dt - g_A(t+\tau)(dt+d\tau) \Rightarrow \\ \frac{d\tau}{dt} &= \frac{g_{\mathrm{F}}(t) - g_A(t+\tau)}{g_A(t+\tau)+\delta_{\mathrm{I}}}. \end{aligned}$$

One might be concerned whether $1+\tau' > 0$, i.e., if $\tau' > -1$. To see that this concern is immaterial take account of the following fact. Because $\beta < 1$, the marginal benefit of adopting technology are infinite so that I can safely assume $g_A > -\delta_{\mathrm{I}}$ (if there was no adoption the two would be equal). Then, I prove by contradiction that $\tau' < -1$ cannot be the case. Suppose $\tau' < -1$. Then,

$$\begin{aligned} \frac{g_{\mathrm{F}} - g_A(t+\tau)}{\delta_{\mathrm{I}} + g_A(t+\tau)} &< -1 && \Rightarrow \\ g_{\mathrm{F}} - g_A(t+\tau) &< -\delta_{\mathrm{I}} - g_A(t+\tau) && \Rightarrow \\ g_{\mathrm{F}} &< -\delta_{\mathrm{I}}. \end{aligned}$$

However, even if there is no research effort whatsoever, the worst frontier growth rate must be at least as high as $-\delta_{\mathrm{I}}$. Thus by contradiction it must be that $\dot{\tau} > -1$.

A.3 Planner Problem

The planner takes M and L_{P} as given. Set up the present value Hamiltonian and solve the program.

$$H = \max_{H_{\mathrm{F}},C} e^{-(\rho-g_L)t}\log(c) + \mu_K\left[(AL_{\mathrm{P}})^{1-\alpha}K^{\alpha} - C - \delta_k K\right] + \mu_A\left[\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}A\right] + \mu_{A_{\mathrm{F}}}\left[\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}A_{\mathrm{F}}\right]$$

$$\begin{aligned} dC: &\qquad \mu_K L = e^{-(\rho-g_L)t}\frac{1}{c} \\ dk: &\qquad -\frac{\dot{\mu}_K}{\mu_K} = \alpha\left(\frac{AL_{\mathrm{P}}}{K}\right)^{1-\alpha} - \delta_k \\ dh_{\mathrm{F}}: &\qquad \frac{\mu_A}{\mu_{A_{\mathrm{F}}}} = \frac{\lambda\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{F}}^{\lambda}}{\beta\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\frac{h_{\mathrm{F}}}{h_{\mathrm{D}}}} \\ dA: &\qquad -\frac{\dot{\mu}_A}{\mu_A} = \frac{\mu_K}{\mu_A}\left(\frac{1-\alpha}{A}\right)(AL_{\mathrm{P}})^{1-\alpha}K^{\alpha} + \left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right] \\ dA_{\mathrm{F}}: &\qquad -\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} = \frac{\mu_A}{\mu_{A_{\mathrm{F}}}}\left[(1-\theta)\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\right] + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}\right] \end{aligned}$$

where the equation (dA) implies a link between the marginal utility of an extra unit of capital (μ_K) and the multiplier (μ_A), and after rearranging

$$-\frac{\dot{\mu}_A}{\mu_A} = \frac{\mu_K L_{\mathrm{P}}}{\mu_A}(1-\alpha)\left(\frac{K}{AL_{\mathrm{P}}}\right)^{\alpha} + \left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right]$$

it becomes clear that in a steady state $\frac{\dot{\mu}_A}{\mu_A} = \frac{\dot{\mu}_K}{\mu_K} + g_L$ since $\left(\frac{K}{AL_{\mathrm{P}}}\right)^{\alpha} + \left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right]$ is stationary, and $\frac{\dot{L}_{\mathrm{P}}}{L_{\mathrm{P}}} = g_L$ as the relative share of labor devoted to production is constant.

The usual link between marginal product of capital, discounting, and per capita consumption growth emerges, which pins down the ratio of physical capital to effective labor used for production (as opposed to entry)

$$\frac{K}{AL_{\mathrm{P}}} = \left(\frac{\alpha}{\rho + g_{\mathrm{F}} + \delta_k}\right)^{\frac{1}{1-\alpha}}.$$

Next, note that $\frac{\dot{\mu}_A}{\mu_A} = \frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}}$. Combining ($dA_F$) with ($dh_{\mathrm{F}}$), and recall $\tilde{\rho} = \rho - g_L$, yields

$$\begin{aligned}
-\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} &= \frac{\mu_A}{\mu_{A_{\mathrm{F}}}}\left[(1-\theta)\,\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\right] + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}\right] \\
\rho - g_L + g_{\mathrm{F}} &= \frac{\mu_A}{\mu_{A_{\mathrm{F}}}}\left[(1-\theta)\,\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\right] + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}\right] \\
\tilde{\rho} + g_{\mathrm{F}} &= \frac{\lambda\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{F}}^{\lambda}}{\beta\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\frac{h_{\mathrm{F}}}{h_{\mathrm{D}}}}\left[(1-\theta)\,\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\right] + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}\right] \\
\tilde{\rho} + g_{\mathrm{F}} &= \frac{\lambda\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{F}}^{\lambda}}{\beta\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\frac{h_{\mathrm{F}}}{h_{\mathrm{D}}}}\left[(1-\theta)\,\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\right] + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}\right] \\
\tilde{\rho} + g_{\mathrm{F}} &= \frac{\lambda\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda}}{\beta\frac{h_{\mathrm{F}}}{h_{\mathrm{D}}}}(1-\theta) + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}\right].
\end{aligned}$$

Using $a_{\mathrm{F}} = \frac{A_{\mathrm{F}}^{1-\phi}}{L}$ and in the steady state $a_{\mathrm{F}} = \frac{\gamma h_{\mathrm{F}}^{\lambda}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}$, and $g_{\mathrm{F}} = \frac{1}{1-\phi}g_L$,

$$\tilde{\rho} + g_{\mathrm{F}} = \frac{\lambda\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right)}{\beta\frac{h_{\mathrm{F}}}{h_{\mathrm{D}}}}(1-\theta) + \left[\phi\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) - \delta_{\mathrm{I}}\right].$$

Rearranging yields the final result

$$\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}} = \frac{\beta}{1-\theta}\cdot\frac{1}{\lambda}\left[\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + (1-\phi)\right].$$

A.4 Planner Problem with Skilled Labor in Production

The planner takes M and L_{P} as given, and chooses the allocation of skilled labor across sectors, and consumption to take account of the inter-temporal dimension of the problem. The difference to the previous setup is that skilled labor is also used in production so setting the skilled labor share in production equal to zero restores the original. I set up the present value Hamiltonian and solve the following program

$$
\begin{aligned}
H \quad &= \max_{H_{\text{F}},C} e^{-\tilde{\rho}t}\log(c) \quad &&+\mu_K\left[\left(\frac{l_{\text{P}}^{1-\eta}h_{\text{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL - c\cdot L - \delta_k K\right] \\
& &&+\mu_A\left[\nu A^{\theta}A_{\text{F}}^{1-\theta}\left(\frac{h_{\text{D}}}{m}\right)^{\beta} - \delta_{\text{I}}A\right] \\
& &&+\mu_{A_{\text{F}}}\left[\gamma A_{\text{F}}^{\phi}Lh_{\text{F}}^{\lambda} - \delta_{\text{I}}A_{\text{F}}\right].
\end{aligned}
$$

The first order conditions read

$$
\begin{aligned}
dc: &\quad \mu_K L = e^{-\tilde{\rho}t}\frac{1}{c} \\
dh: &\quad \mu_k\frac{\eta(1-\alpha)}{h_{\text{p}}}\left(\frac{l_{\text{P}}^{1-\eta}h_{\text{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL = \mu_A\frac{\beta\nu A^{\theta}A_{\text{F}}^{1-\theta}}{h_{\text{D}}}\left(\frac{h_{\text{D}}}{m}\right)^{\beta} = \mu_{A_{\text{F}}}\frac{\lambda\gamma A_{\text{F}}^{\phi}Lh_{\text{F}}^{\lambda}}{h_{\text{F}}}
\end{aligned}
$$

and the optimality conditions associated with the co-state variables read

$$
\begin{aligned}
dC: &\quad \mu_K L = e^{-\tilde{\rho}t}\frac{1}{c} \\
dh: &\quad \mu_K\frac{\eta(1-\alpha)}{h_{\text{p}}}\left(\frac{l_{\text{P}}^{1-\eta}h_{\text{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL = \mu_A\frac{\beta\nu A^{\theta}A_{\text{F}}^{1-\theta}}{h_{\text{D}}}\left(\frac{h_{\text{D}}}{m}\right)^{\beta} = \mu_{A_{\text{F}}}\frac{\lambda\gamma A_{\text{F}}^{\phi}Lh_{\text{F}}^{\lambda}}{h_{\text{F}}} \\
dk: &\quad -\dot{\mu}_K = \mu_K\left[\alpha\left(\frac{l_{\text{P}}^{1-\eta}h_{\text{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}k^{\alpha-1} - \delta_k\right] \text{ (with } k := \frac{K}{AL}\text{)} \\
dA: &\quad -\dot{\mu}_A = \mu_K\left(\frac{1-\alpha}{A}\right)\left(\frac{l_{\text{P}}^{1-\eta}h_{\text{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL + \mu_A\left[\theta\nu A^{\theta-1}A_{\text{F}}^{1-\theta}\left(\frac{h_{\text{D}}}{m}\right)^{\beta} - \delta_{\text{I}}\right] \\
dA_{\text{F}}: &\quad -\dot{\mu}_{A_{\text{F}}} = \mu_A\left[(1-\theta)\nu A^{\theta}A_{\text{F}}^{-\theta}\left(\frac{h_{\text{D}}}{m}\right)^{\beta}\right] + \mu_{A_{\text{F}}}\left[\phi\gamma A_{\text{F}}^{\phi-1}Lh_{\text{F}}^{\lambda} - \delta_{\text{I}}\right].
\end{aligned}
$$

Equation (dA) implies a link between the marginal utility of an extra unit of capital (μ_K) and the multiplier (μ_A), and after rearranging

$$-\frac{\dot{\mu}_K}{\mu_K} = \alpha\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha} k^{\alpha-1} - \delta_k \tag{62}$$

$$-\frac{\dot{\mu}_A}{\mu_A} = \frac{\mu_K}{\mu_A}(1-\alpha)\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha} L + \left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right] \tag{63}$$

$$-\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} = \frac{\mu_A}{\mu_{A_{\mathrm{F}}}}\left[(1-\theta)\,\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\right] + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}}\right]. \tag{64}$$

Focus first on the case with exogenous innovation, so I can ignore all terms that involve the co-state variable $\mu_{A_{\mathrm{F}}}$. Note that from (dc) I have $-\frac{\dot{\mu}_K}{\mu_K} = \rho + g_{\mathrm{F}}$ along a balanced growth path. Next, note that the only way for $\frac{\dot{\mu}_A}{\mu_A}$ in (63) to be constant is for $\frac{\dot{\mu}_A}{\mu_A} = \frac{\dot{\mu}_K}{\mu_K} + g_L$ to be true. This implies

$$\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} = \frac{\mu_K}{\mu_A}(1-\alpha)\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha} L + \theta\nu z^{\theta-1}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}$$

$$\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} = \frac{\mu_K}{\mu_A}(1-\alpha)\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha} L + \theta\,(g_{\mathrm{F}} + \delta_{\mathrm{I}})$$

$$\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} = \frac{\mu_K}{\mu_A}(1-\alpha)\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha} L + \theta\,(g_{\mathrm{F}} + \delta_{\mathrm{I}}) \tag{65}$$

where the second lines uses the link between h_{D} and z in the steady state. Next, note

$$\frac{\mu_k}{\mu_A}\frac{\eta\,(1-\alpha)}{h_{\mathrm{p}}}\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha} L = \frac{\beta\nu z^{\theta-1}}{h_{\mathrm{D}}}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}$$

$$\frac{\mu_k}{\mu_A} = \frac{h_{\mathrm{P}}}{h_{\mathrm{D}}}\frac{\beta\,(g_{\mathrm{F}} + \delta_{\mathrm{I}})}{\eta\,(1-\alpha)\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha} L}$$

which can be used in (65) to get

$$\frac{h_{\mathrm{D}}}{h_{\mathrm{P}}} = \frac{\beta}{1-\theta}\frac{1}{\eta}\left\{\frac{1}{1+\frac{\tilde{\rho}}{(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}}\right\}$$

which characterizes the efficient allocation of skilled labor in the case without innovation but with skilled labor as a factor of production in the intermediates good sector.

Next, suppose firms are automatically at the technological frontier, and set μ_A to zero while $z = 1$.

This scenario is almost identical with Romer (1990). In that case,

$$
\begin{aligned}
dh: &\qquad \mu_{A_{\mathrm{F}}} \frac{\lambda \gamma A_{\mathrm{F}}^{\phi} L h_{\mathrm{F}}^{\lambda}}{h_{\mathrm{F}}} = \mu_k \frac{\eta (1-\alpha)}{h_{\mathrm{p}}} \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{AL} \right)^{\alpha} AL \\
dA_F: &\qquad -\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} = \frac{\mu_K}{\mu_{A_{\mathrm{F}}}} (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{A_{\mathrm{F}} L} \right)^{\alpha} L + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}} \right].
\end{aligned}
$$

The same logic as before implies $\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} = \frac{\dot{\mu}_K}{\mu_k} + g_L$ so

$$
\begin{aligned}
\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} &= \frac{\mu_K}{\mu_{A_{\mathrm{F}}}} (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{A_{\mathrm{F}} L} \right)^{\alpha} L + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{F}}^{\lambda} - \delta_{\mathrm{I}} \right] \\
\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} &= \frac{\gamma L}{A_{\mathrm{F}}^{1-\phi}} h_{\mathrm{F}}^{\lambda} \left(\lambda \frac{h_{\mathrm{p}}}{h_{\mathrm{F}}} \frac{1}{\eta} + \phi \right).
\end{aligned}
$$

Now using $a_{\mathrm{F}} = \frac{A_{\mathrm{F}}^{1-\phi}}{L}$ and $\gamma h_{\mathrm{F}}^{\lambda} = a_{\mathrm{F}} (g_{\mathrm{F}} + \delta_{\mathrm{I}})$ I get the ratio of skilled labor in production vis-a-vis innovation.

$$
\frac{h_{\mathrm{p}}}{h_{\mathrm{F}}} = \left\{ \frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + 1 - \phi \right\} \frac{\eta}{\lambda}.
$$

This expression is extremely similar to the result in Jones (1995).

Taking out inefficiencies in research and adoption. First, using planner and private solution note

$$
\begin{aligned}
\frac{\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{SP}}}{\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{DC}}} &= \frac{\frac{\beta}{1-\theta} \frac{1}{\lambda} \left\{ \frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + (1-\phi) \right\}}{\frac{\beta}{1-\theta} \frac{1}{\alpha} \left\{ \frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + 1 \right\} \frac{z^{-\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}}{1 + \frac{\rho + \delta_{\mathrm{X}}}{(g_{\mathrm{F}} + \delta_{\mathrm{I}})(1-\theta)}}} \Rightarrow \\
\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{SP}} &= \left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{DC}} \frac{\alpha}{\lambda} \frac{\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + (1-\phi)}{\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + 1} z^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} \left(1 + \frac{\rho + \delta_{\mathrm{X}}}{(g_{\mathrm{F}} + \delta_{\mathrm{I}})(1-\theta)} \right),
\end{aligned}
$$

which, after using $\alpha = \lambda$, $\phi = 0$, and $\delta_{\mathrm{X}} = -g_L$, collapses to

$$
\left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{SP}} = \left(\frac{h_{\mathrm{D}}}{h_{\mathrm{F}}}\right)^{\mathrm{DC}} \underbrace{z^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}}_{\leq 1} \left(1 + \underbrace{\frac{\tilde{\rho}}{(1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})}}_{\geq 1} \right).
$$

The bias is ambiguous and I conjecture that search externalities in either activity play a role. It would be desirable to understand this results more carefully.

A.5 Open Economy

Derivation of the share of ideas originating from country k.

You start with the free entry condition into research, and because the value of innovation is the same no matter where you innovate (because ideas are sold to the same world market frictionlessly), and assuming all countries innovate (which is always true for $\lambda < 1$), I have

$$\frac{\left(h_{\mathrm{F},c}\right)^{1-\lambda} w_{H,c}}{\gamma_c \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi}} = \frac{\left(h_{\mathrm{F},k}\right)^{1-\lambda} w_{H,k}}{\gamma_k \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi}} \Rightarrow \tag{66}$$
$$\frac{\left(h_{\mathrm{F},c}\right)^{1-\lambda} w_{H,c}}{\gamma_c} = \frac{\left(h_{\mathrm{F},k}\right)^{1-\lambda} w_{H,k}}{\gamma_k}.$$

Next, use the fact that by the resource constraint, a link between share of ideas and skilled labor devoted to innovation emerges

$$\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) \chi_c a_{\mathrm{F}}^{\mathrm{W}} = \gamma_c h_{\mathrm{F},c}^{\lambda}.$$

Substituting this in (66) and rearranging yields

$$\frac{\left(\frac{\chi_c}{\gamma_c}\right)^{\frac{1-\lambda}{\lambda}} w_{H,c}}{\gamma_c} = \frac{\left(\frac{\chi_k}{\gamma_k}\right)^{\frac{1-\lambda}{\lambda}} w_{H,k}}{\gamma_k} \Rightarrow$$
$$\frac{\chi_c}{\chi_k} = \left(\frac{\gamma_c}{\gamma_k}\right)^{\frac{1}{1-\lambda}} \left(\frac{w_{H,c}}{w_{H,k}}\right)^{-\frac{\lambda}{1-\lambda}}.$$

Next, note that the high-skilled wage ratio is a function of the skill premium alone $\frac{w_{H,k}}{w_{H,c}} = \left(\frac{s_i}{s_c}\right)^{\frac{1-(\beta+\theta)}{1-\theta}}$. Summing over all k implies

$$\sum_c \frac{\chi_c}{\chi_k} = \sum_c \left(\frac{\gamma_c}{\gamma_k}\right)^{\frac{1}{1-\lambda}} \left(\frac{w_{H,c}}{w_{H,k}}\right)^{-\frac{\lambda}{1-\lambda}}$$

$$\sum_c \frac{\chi_c}{\chi_k} = \sum_c \left(\frac{\gamma_c}{\gamma_k}\right)^{\frac{1}{1-\lambda}} \left(\left(\frac{s_c}{s_k}\right)^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}$$

$$\frac{1}{\chi_k} = \frac{\sum_c \gamma_c^{\frac{1}{1-\lambda}} \left(s_c^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}}{\gamma_k^{\frac{1}{1-\lambda}} \left(s_k^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}} \Rightarrow$$

$$\chi_k = \frac{\gamma_k^{\frac{1}{1-\lambda}} \left(s_k^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} \left(s_c^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}}.$$

This establishes the result in the main part of the paper.

Derivation of the key market clearing condition in the open economy.

$$\frac{w_{H,k}}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha L_{\mathrm{P}}}{\left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{1-\phi}} \sum_c w_c z_c^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$\frac{w_{H,k}}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{\left(a_{\mathrm{F}}^{\mathrm{W}}\right)} \sum_c w_c z_c^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$\frac{w_{H,k}}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} = \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{\gamma_k h_{\mathrm{F},k}^{\lambda}} \chi_k \sum_c w_c z_c^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$z_k s_k h_{\mathrm{F}} = \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \alpha l_{\mathrm{P}} \chi_k \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$z_k s_k h_{\mathrm{F},k} = \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \alpha l_{\mathrm{P}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}.$$

Note that $z = s^{-\frac{\beta}{1-\theta}} \kappa_z$, same as in the closed economy, and collect terms that are constant along a balanced growth path so

$$h_{\mathrm{F},k} = \Lambda_{\mathrm{FO}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}.$$

Combine this with the market clearing condition

$$h_{\text{tot},k} = \frac{\Lambda_{\text{D}}}{s_k} + \Lambda_{\text{FO}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A+\delta_{\text{I}}}}.$$

Generalized ACR Formula. Recall the expression for real wages in the open economy relative to the closed economy reads

$$\frac{w_k^{open}}{w_k^{closed}} = \left(\frac{h_{\text{F},k}^{open}}{h_{\text{F},k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}} \cdot \left(\frac{s_k^{open}}{s_k^{closed}}\right)^{-\frac{\beta}{1-\theta}}$$

and

$$\frac{w_H^{open}}{w_H^{closed}} = \left(\frac{h_{\text{F},k}^{open}}{h_{\text{F},k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}} \cdot \left(\frac{s_k^{open}}{s_k^{closed}}\right)^{\frac{1-(\beta+\theta)}{1-\theta}}.$$

To derive this expression, start with the resource constraint which implies a link between equilibrium research effort and the share of ideas invented by country k

$$a_{\text{F}}^{\text{W}} = \frac{\gamma_k h_{\text{F},k}^{\lambda}}{(g_{\text{F}} + \delta_{\text{I}})\,\chi_k}.$$

Now the ratio of frontier technology in the open and closed economy is given by $\frac{A_{\text{F}}^{\text{W},open}}{A_{\text{F}}^{\text{W},closed}} = \left(\frac{a_{\text{F}}^{\text{W},open}}{a_{\text{F}}^{\text{W},closed}}\right)^{\frac{1}{1-\phi}} = \left(\frac{h_{\text{F},k}^{open}}{h_{\text{F},k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}}$ where I used the fact that $\chi_k^{closed} = 1$.

To study the real wage effects I have to account for the adoption margin since $\frac{w_k^{open}}{w_k^{closed}} = \frac{A_{\text{F}}^{\text{W},open}}{A_{\text{F}}^{\text{W},closed}} \frac{z_k^{open}}{z_k^{closed}}$. Note that $\frac{z_k^{open}}{z_k^{closed}} = \left(\frac{s_k^{open}}{s_k^{closed}}\right)^{-\frac{\beta}{1-\theta}}$, which delivers the result. For skilled wages the skill premium needs to be added, $\frac{w_H^{open}}{w_H^{closed}} = \frac{A_F^{W,open}}{A_F^{W,closed}} \frac{z^{open}}{z^{closed}} \frac{s^{open}}{s^{closed}}$.

These wage ratios reflect the long-run differences in wages after all temporary adjustments have taken place. In particular, since the long-run supply of capital is perfectly elastic, the capital-effective labor ratio is the same in the open and closed economy and is thus netted out in the ratio. This concludes the derivation.

Open Economy with country size differences. I next generalize the framework to allow for countries of heterogenous sizes, which helps me to take the framework to the data. I use the "home economy" aka the West as baseline for the normalizations. This means that the definition of the normalized world technological frontier is unchanged. Define $b_c = \frac{L_c}{L} \in R^+$ as a weight attached to country c relative to the home economy. You could as well pick $L_{\text{W}} = \sum_c L_c$ or any other normalizing factor that grows as the same rate as the labor force.

The growth of the technological frontier now reads

$$\begin{aligned}
\frac{\dot{A}_{\mathrm{F}}^{\mathrm{W}}}{A_{\mathrm{F}}^{\mathrm{W}}} &= \sum_c \frac{\dot{A}_{\mathrm{F},c}}{A_{\mathrm{F}}^{\mathrm{W}}} \\
&= \sum_c \chi_c \frac{\dot{A}_{\mathrm{F},c}}{A_{\mathrm{F},c}} \\
&= \sum_c \chi_c \left(\frac{\gamma_c}{\chi_c} \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi-1} L_c h_{\mathrm{F},c}^{\lambda} - \delta_{\mathrm{I}} \right) \\
&= \sum_c \chi_c \left(\frac{\gamma_c}{\chi_c} \frac{1}{a_{\mathrm{F}}^{\mathrm{W}}} h_{\mathrm{F},c}^{\lambda} b_c - \delta_{\mathrm{I}} \right) \\
&= \sum_c \chi_c \left(\frac{\gamma_c}{\chi_c} \frac{h_{\mathrm{F},c}^{\lambda} b_c}{a_{\mathrm{F}}^{\mathrm{W}}} - \delta_{\mathrm{I}} \right) \\
&= \sum_c \frac{\gamma_c h_{\mathrm{F},c}^{\lambda} b_c}{a_{\mathrm{F}}^{\mathrm{W}}} - \delta_{\mathrm{I}}.
\end{aligned}$$

This result of course coincides with the previous result when countries are equal-sized and $b_c = 1$. Consequently,

$$g_{a_{\mathrm{F}}^{\mathrm{W}}} = (1-\phi) \left\{ \sum_c \frac{\gamma_c h_{\mathrm{F},c}^{\lambda} b_c}{a_{\mathrm{F}}^{\mathrm{W}}} - \left(\delta_{\mathrm{I}} + \frac{1}{1-\phi} g_L \right) \right\}$$

and in the steady state

$$a_{\mathrm{F}}^{\mathrm{W}} = \frac{\sum_c \gamma_c h_{\mathrm{F},c}^{\lambda} b_c}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}.$$

By free entry, I have

$$\frac{h_{\mathrm{F},c}^{1-\lambda} w_{H,c}}{\gamma_c} = \frac{h_{\mathrm{F},k}^{1-\lambda} w_{H,k}}{\gamma_k}$$

and by the resource constraint I have

$$\frac{\chi_c}{\chi_k} = \frac{\gamma_c h_{\mathrm{F},c}^{\lambda} b_c}{\gamma_k h_{\mathrm{F},k}^{\lambda} b_k}.$$

Combining free entry and resource constraint delivers

$$\chi_k = \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_k}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_c}$$

where I used $w_{H,k} = b_t s^{\frac{1-\theta-\beta}{1-\theta}}$.

I next derive the demand for skilled labor as before, using

$$
\begin{aligned}
\frac{s_k}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} &= \frac{1}{w_k} \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha}{\left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{1-\phi}} \sum_c L_{\mathrm{P},c} w_c z_c^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} \\
\frac{s_k}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} &= \frac{1}{w_k} \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}^{\mathrm{W}}} \sum_c w_c z_c^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} \frac{L_c}{L} \\
\frac{s_k z_k}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} &= \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}^{\mathrm{W}}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c \\
\frac{s_k z_k}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} &= \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}^{\mathrm{W}}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c .
\end{aligned}
$$

Next, use $\chi_k = \frac{\gamma_k h_{\mathrm{F},k}^{\lambda} b_k}{a_{\mathrm{F}}^{\mathrm{W}}(g_{\mathrm{F}} + \delta_{\mathrm{I}})}$ to obtain

$$
\begin{aligned}
\frac{s_k z_k}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} &= \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}^{\mathrm{W}}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c \\
\frac{s_k z_k}{\gamma_k} h_{\mathrm{F},k}^{1-\lambda} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\chi_k \alpha l_{\mathrm{P}}}{\gamma_k h_{\mathrm{F},k}^{\lambda} b_k} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c \\
s_k z_k h_{\mathrm{F},k}^{1-\lambda} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\chi_k \alpha l_{\mathrm{P}}}{h_{\mathrm{F},k}^{\lambda} b_k} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c \\
s_k z_k h_{\mathrm{F},k} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_k}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_c} \alpha l_{\mathrm{P}} \frac{1}{b_k} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c \\
s_k z_k h_{\mathrm{F},k} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_c} \alpha l_{\mathrm{P}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c \\
\kappa_z s_k^{\frac{1-\theta-\beta}{1-\theta}} h_{\mathrm{F},k} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_c} \alpha l_{\mathrm{P}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c \\
h_{\mathrm{F},k} &= \Lambda_{\mathrm{FO}} \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_c} \alpha l_{\mathrm{P}} \sum_c z_c^{1+\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_c .
\end{aligned}
$$

Note that for the symmetric country case with all endogenous variables and exogenous productivity shifters identical across countries, the expression would coincide with the solution to the closed economy setup since the term $\sum_c b_c$ appears in both nominator and denominator.

Inserting this expression into the market clearing condition and iterating over a set of skill premia across countries delivers the long-run steady state allocation

$$\frac{\Lambda_{\mathrm{D}}}{s_k}+\Lambda_{\mathrm{FO}}\frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}}\frac{\gamma_k^{\frac{1}{1-\lambda}}s_k^{-\frac{1}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}}}{\sum_c\gamma_c^{\frac{1}{1-\lambda}}s_c^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}}b_c}\alpha l_{\mathrm{P}}\sum_c z_c^{1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}b_c=h_{\mathrm{tot},k}.$$

A.5.1 Trade cost and bargaining in open economy

I will next show to to extend the open economy model to include trade cost and bargaining. These extensions are potentially important in taming the impact of the East on innovation in the West. In a model where innovation happens in the West, and the East is a relatively large country that adopts Western technology, the pull force can become so strong that the skill premium in the West shoots up so much that Western workers are left impoverished. Two reasonable assumptions to avoid this outcome relate to the role of trade cost, and bargaining over innovator rents, which I consider in turn.
Trade cost. Recall the flow profits of innovation in the steady state are proportional to

$$\pi_{\mathrm{I}}\propto\sum_c\frac{\alpha L_c^P w_c z^{\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}}{A_F z_c}.$$

One could easily introduce ice berg trade cost from innovator i to technology user c

$$\pi_{\mathrm{I}}\propto\sum_c\frac{1}{\tau_{ic}}\frac{\alpha L_c^P w_c z^{\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}}{A_F z_c}$$

where $\tau_{ic}\geq 1$ and $\tau_{ii}=1$.

The trade cost is somewhat non standard because the model differs from, for instance, from Krugman (1980) where the trade elasticity $\sigma-1$ shapes the importance of trade cost. The reason is that the only good that is traded is the royalty associated with innovator profits. The locally optimal markup is applied onto domestic cost, which maximizes total domestic revenues, and there is no way to further raise profits. However, the profits need to be shipped back to the holder of the patents, and trade costs apply so only a fraction $\frac{1}{\tau_{ij}}$ of the value of the idea arrive at home.

One could envision that financial markets can help overcome this trade cost when both countries hold ideas. That is, instead of shipping the royalty back one could simply trade it for the royalty that accrues to foreign firms in the domestic economy. The extent to which this is possible depends on the degree of specialization and would not work if all ideas are held in the West.
Bargaining over royalties. An alternative interpretation for the wedge parameter τ could be microfounded by considering explicit bargaining over innovator profits. For example, one could imagine

that the Chinese government could try to extract rents from foreign innovators' ideas. This would help reduce the impact of foreign adoption on innovation in the West. If the (corrupt) government wastes this income, the model coincides exactly with the previous extension. Alternatively, one can rebate the profits to households.

Elastic skill supply. I reiterate that generalizing the supply of skilled labor, or using both high skilled and production labor in innovation, would help deal with the impact of a large emerging market on Western adoption by softening the blow to production workers.

A.5.2 Total Asset Stock in Open Economy

I derive the total value of assets in the open economy, which depends on the value of domestic capital, domestic production sector firms, and global intellectual property. Note that ideas are adopted at different points in time in different countries, which I need to keep track off. I focus on the two-country case, and I focus on the value of an idea, which is the complicated object when constructing the total value of assets held by the home economy. I focus on the case where all intellectual property is held in the home economy (West).

It is easy to see that the value of an innovation $V_{\mathrm{I}}^{\mathrm{W}}$ can be split into home and foreign component

$$V_{\mathrm{I}}^{\mathrm{W}} = V_{\mathrm{I}} + V_{\mathrm{I}}^{*}.$$

Next, note that along a balanced growth path a simple expression for the value of an innovation of ideas that are already adopted, denoted by V_A, obtains

$$\begin{aligned} V_A &= \frac{1}{\rho - g_L + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha L_{\mathrm{P}} w_t}{A} \\ V_A^{*} &= \frac{1}{\rho - g_L + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha L_{\mathrm{P}}^{*} w_t^{*}}{A^{*}}. \end{aligned}$$

Conveniently, changes in the real wage for a fixed long-run interest rate have no bearing on the value of an innovation since they directly cancel with the total numbers of adopted ideas in the denominator (recall $w \propto A$).

The forces that do matter are i) changes in the waiting time, and ii) an extensive margin effect in the form of another country using ideas. I focus on the first issue first, and introduce the following notation. Let $V_{\mathrm{I},t}(k)$ denote the value of an innovation at time t that is k years away from adoption. Clearly, this asset has value, but its value ought to be below ideas that are already in use at time t.

Using the HJB equation $(r+\delta_{\mathrm{I}})\,V_{\mathrm{I}} = 0 + \dot{V}_{\mathrm{I}}$, and noting $V_{\mathrm{I},t+k}(0) = V_{A,t+k}$, I have

$$\begin{aligned} V_{\mathrm{I},t}(k) &= V_{\mathrm{I},t+k}(0)\, e^{-(r+\delta_{\mathrm{I}})k} \\ &= V_{A,t+k} e^{-(r+\delta_{\mathrm{I}})k} \\ &= V_{A,t} e^{-(r+\delta_{\mathrm{I}}-g_L)k} \end{aligned}$$

where the last line follows from using $\frac{V_{A,t+k}}{V_{A,t}} = e^{g_L k}$.

Next, note that if we knew the distribution of k, i.e., the waiting time of non-adopted ideas, we could already compute the total value of intellectual property using

$$A_t \cdot V_{A,t} + (A_{\mathrm{F},t} - A_t) \cdot \int_0^{\tau} V_{\mathrm{I},t}(k)\, dF(k) + A_t^* \cdot V_{A,t}^* + (A_{\mathrm{F},t} - A_t^*) \cdot \int_0^{\tau^*} V_{\mathrm{I},t}(k^*)\, dF(k^*)$$

where $dF(k)$ is the appropriate marginal distribution over wait times ranging from zero to τ.

The distribution of k can be derived as follows. Note that the share of ideas close to adoption among all unadopted ideas depends on how much entry there is, which is directly proportional to the gross entry rate $g_{\mathrm{F}} + \delta_{\mathrm{I}}$. In fact, the distribution k follows a truncated exponential distribution

$$f(k) = \frac{(g_{\mathrm{F}} + \delta_{\mathrm{I}})\, e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})k}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1}, k \in [0, \tau],$$

where the distribution in the foreign economy features a different τ. Putting the pieces together, and focusing on the value of innovation in the advanced economy, yields

$$
\begin{aligned}
A_t \cdot V_{A,t} + (A_{\mathrm{F},t} - A_t) \cdot \int_0^\tau V_{\mathrm{I},t}(k)\, dF(k) &= A_t \cdot V_{A,t} \left(1 + \left(\frac{A_{\mathrm{F},t}}{A_t} - 1\right) \cdot \int_0^\tau \frac{V_{\mathrm{I},t}(k)}{V_{A,t}} dF(k)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \cdot \int_0^\tau \frac{V_{\mathrm{I},t}(k)}{V_{A,t}} dF(k)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \cdot \int_0^\tau e^{-(r+\delta_{\mathrm{I}}-g_L)k} dF(k)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \cdot \int_0^\tau e^{-(r+\delta_{\mathrm{I}}-g_L)k} \frac{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\, e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})k}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} dk\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot \int_0^\tau e^{-(r-g_{\mathrm{F}}-g_L)k} dk\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot \int_0^\tau e^{-(\rho-g_L)k} dk\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot (\rho - g_L) \left[e^{-(\rho-g_L)k}\right]_\tau^0\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{1 - e^{-(\rho-g_L)\tau}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot (g_{\mathrm{F}}+\delta_{\mathrm{I}})(\rho - g_L)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{1 - z^{\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}}{\frac{1-z}{z}} \cdot (g_{\mathrm{F}}+\delta_{\mathrm{I}})(\rho - g_L)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(1 - z^{\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}\right) \cdot (g_{\mathrm{F}}+\delta_{\mathrm{I}})(\rho - g_L)\right)
\end{aligned}
$$

where I substituted out $\tau = -\frac{\log z}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}$. Now adding the value of intellectual property across both countries yields

$$
Tot\_pat\_value = \frac{\alpha L_{\mathrm{P}} w}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \left\{ \left(1 + \left(1 - z^{\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}\right) \cdot (g_{\mathrm{F}}+\delta_{\mathrm{I}})(\rho - g_L)\right) + \underbrace{\frac{z^* b^*}{z} \left(1 + \left(1 - {z^*}^{\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}\right) \cdot (g_{\mathrm{F}}+\delta_{\mathrm{I}})(\rho - g_L)\right)}_{\text{value of patents abroad}} \right\}
$$

where b^* keeps track of the relative country size $b^* = \frac{L_{\mathrm{P}}^*}{L_{\mathrm{P}}}$. Using this expression it is easy to compare the total value of ideas to the counterfactual autarky equilibrium. In the counterfactual autarky equilibrium, the value of patents abroad would be gone, and I evaluate adoption gap and wages at the counterfactual autarky level.

Next, I add up all assets in the economy and normalize by L to get

$$\begin{aligned} Assets &= \frac{Tot\_pat\_value}{L} + \frac{K}{L} + mV_M \\ &= w\left(\frac{Tot\_pat\_value}{Lw} + \frac{\alpha^2}{1-\alpha}\frac{l_P}{\rho + g_{\mathrm{F}} + \delta_K} + m \cdot f_{\mathrm{E}}\right). \end{aligned}$$

I can construct this expression and compare it conveniently across different counterfactual scenarios where all elements inside the parentheses are stationary. Lastly, note that one can simply apply the factor χ to the value of patents when both countries are innovation, and one can use the wage of the emerging market as normalizing factor to perform the same analysis.

A.5.3 Computing the Effect of an Increase in Research Productivity on Skill Premia.

I show how to compute the effect of an increase in a country's research productivity on skill premia everywhere. Consider the case with equal-sized production labor endowments across countries. Start with the market clearing condition and simplify without loss of generality by setting $\tilde{\gamma}_k = \gamma_k^{\frac{1}{1-\lambda}}$,

$$h_{k,\mathrm{tot}} = \frac{\Lambda_{\mathrm{D}}}{s_k} + \frac{\Lambda_{\mathrm{FO}}\tilde{\gamma}_k s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_c \tilde{\gamma}_c s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_c s_c^{-\frac{\beta}{1-\theta}\left(1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}\right)}. \tag{67}$$

Differentiate (67) respect to the set of endogenous skill premia s and exogenous technology shifter $\tilde{\gamma}_k = \gamma_k^{\frac{1}{1-\lambda}}$ (I change only this one technology parameter so $d\tilde{\gamma}_c = 0\,\forall c \neq k$) follows from noting

$$\begin{aligned} d\left(\frac{\Lambda_{\mathrm{D}}}{s_k}\right) &= -\frac{\Lambda_{\mathrm{D}}}{s_k} d\log s_k \\ d\left(\sum_c s_c^{-\kappa}\right) &= -\kappa \sum_c s_c^{-\kappa} d\log s_c \\ d\left(\frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}}}{\sum_c \tilde{\gamma}_c s_c^{-\delta}}\right) &= -\frac{\delta}{\lambda}\frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}}}{\sum_c \tilde{\gamma}_c s^{-\delta}} d\log s_k + \delta \frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}} \sum_c \tilde{\gamma}_c s_c^{-\delta} d\log s_c}{\left(\sum_c \tilde{\gamma}_c s_c^{-\delta}\right)^2} \\ &\quad + \frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}}}{\sum_c \tilde{\gamma}_c s_c^{-\delta}} d\log\tilde{\gamma} - \frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}}}{\sum_c \tilde{\gamma}_c s_c^{-\delta}} \frac{\tilde{\gamma}_k s_k^{-\delta} d\log\tilde{\gamma}}{\sum_c \tilde{\gamma}_c s_c^{-\delta}} \\ &= -\frac{\delta}{\lambda}\frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}}}{\sum_c \tilde{\gamma}_c s^{-\delta}} d\log s_k + \delta \frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}}}{\sum_c \tilde{\gamma}_c s_c^{-\delta}} \sum_c \chi_c d\log s_c + (1-\chi_k)\frac{\tilde{\gamma}_k s_k^{-\frac{\delta}{\lambda}}}{\sum_c \tilde{\gamma}_c s_c^{-\delta}} d\log\tilde{\gamma} \end{aligned}$$

where $\delta = \frac{1-\theta-\beta}{1-\theta}\frac{\lambda}{1-\lambda}$ and $\kappa = \frac{\beta}{1-\theta}\left(1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}\right)$. Putting the pieces together and totally differentiating the market clearing, after using $dh_{c,\text{tot}} = 0$ since the relative skill share is fixed, yields

$$\begin{aligned} 0 = &-h_{\mathrm{D},k} d\log s_k - h_{\mathrm{F},k}\frac{\delta}{\lambda} d\log s_k \\ &+ \delta h_{\mathrm{F},k}\sum_c \chi_c d\log s_c - \kappa h_{\mathrm{F},k}\sum_c \frac{s_c^{-\kappa}}{\sum_c s_c^{-\kappa}} d\log s_c \\ &+ h_{\mathrm{F},k}\left(1-\chi_k\right) d\log\tilde{\gamma} \end{aligned}$$

where I substituted out $\frac{\Lambda_{\mathrm{FO}}\gamma^{\frac{1}{1-\lambda}} s^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_c \gamma_c^{\frac{1}{1-\lambda}} s_c^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}}\sum_c s_c^{-\frac{\beta}{1-\theta}\left(1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}\right)} = h_{\mathrm{F},c}$.

Rearranging and noting that I only shock the research productivity of country k leads to

$$d\log s_k = \frac{\delta h_{\mathrm{F},k}}{h_{\mathrm{D},k}+h_{\mathrm{F},k}\frac{\delta}{\lambda}}\sum_c \chi_c d\log s_c - \frac{\kappa h_{\mathrm{F},k}}{h_{\mathrm{D},k}+h_{\mathrm{F},k}\frac{\delta}{\lambda}}\sum_c \frac{s_c^{-\kappa}}{\sum_j s_j^{-\kappa}} d\log s_c + \frac{h_{\mathrm{F},k}}{h_{\mathrm{D},k}+h_{\mathrm{F},k}\frac{\delta}{\lambda}}\left(1-\chi_k\right) d\log\tilde{\gamma}_k$$

for country k, while for any other country I have

$$d\log s_j = \frac{\delta h_{\mathrm{F},j}}{h_{\mathrm{D},j}+h_{\mathrm{F},j}\frac{\delta}{\lambda}}\sum_c \chi_c d\log s_c - \frac{\kappa h_{\mathrm{F},j}}{h_{\mathrm{D},j}+h_{\mathrm{F},j}\frac{\delta}{\lambda}}\sum_c \frac{s_c^{-\kappa}}{\sum_j s_j^{-\kappa}} d\log s_c - \frac{h_{\mathrm{F},j}}{h_{\mathrm{D},j}+h_{\mathrm{F},j}\frac{\delta}{\lambda}}\chi_k d\log\tilde{\gamma}_k.$$

I few observations are noteworthy. First, a country $j's$ exposure to an improvement in research productivity in k hinges on whether country j performs frontier research, i.e., whether $h_{\mathrm{F},j} > 0$. This happens for two reasons. First, improving research productivity of a foreign country reduce the home economy's relative competitiveness in innovation, and causes a decline in research activity, captures in the term $h_{\mathrm{F},j}\chi_k$. This effect also matters indirectly through $\frac{\delta h_{\mathrm{F},j}}{h_{\mathrm{D},j}+h_{\mathrm{F},j}\frac{\delta}{\lambda}}\sum_c \chi_c d\log s_c$, i.e., changes in foreign skill premium impact the domestic economy's specialization in innovation, which in turn impacts their own skill premium. Second, an additional effect emerges due to the technology adoption margin: as skill premia change, technology adoption changes, which in turn impacts the returns to innovation.

To solve the system I rewrite it in Matrix form

$$d\log\mathbf{s} = [\mathbf{A}(\mathbf{s}) - \mathbf{B}(\mathbf{s})]d\log\mathbf{s} + \mathbf{C}(\chi)d\log\tilde{\gamma} \tag{68}$$

where small bold letters are vectors and large bold letters denote matrices.

One can further simplify this system by noting that matrix $\mathbf{A}$ and $\mathbf{B}$ can be written as the product of two vectors. Define $\zeta_j := \frac{\kappa h_{\mathrm{F},j}}{h_{\mathrm{D},j}+h_{\mathrm{F},j}\frac{\delta}{\lambda}}$, and $\pi_j := \frac{\delta}{\kappa}\chi_j - \frac{s_j^{-\kappa}}{\sum_j s_j^{-\kappa}}$ where π_j is bounded, i.e., $\pi_j \in (-1, \frac{\delta}{\kappa}]$.

Using two vectors $\zeta, \pi \in R^{1\times N}$, the system can be written as

$$d\log \mathbf{s} = [\zeta\pi']d\log \mathbf{s} + \mathbf{C}(\chi)d\log\tilde{\gamma} \tag{69}$$

which is convenient as the solution to (69) follows from computing the inverse of $\mathbb{I} - \zeta\pi'$ where $\mathbb{I}$ is the identity matrix. This solution can be derived in closed form using the Sherman-Morrison formula

$$\left(\mathbb{I} - \zeta\pi'\right)^{-1} = \mathbb{I}^{-1} + \frac{\mathbb{I}^{-1}\zeta\pi'\mathbb{I}^{-1}}{1 + \pi'\mathbb{I}^{-1}\zeta}$$
$$\left(\mathbb{I} - \zeta\pi'\right)^{-1} = \mathbb{I} + \frac{\zeta\pi'}{1 + \pi'\zeta}$$

where a sufficient condition for this inverse to be well-defined is for $|\pi'\zeta| < 1$, i.e., the sum should lie within the unit circle.[89] Since $0 < \sum_j \zeta_j \frac{\delta}{\kappa}\chi_j \leq \sum_j \chi_j = 1$, one only has to show that $-\sum_j \frac{s_j^{-\kappa}}{\sum_j s_j^{-\kappa}}\zeta_j \geq -1$. Since $-\sum_j \frac{s_j^{-\kappa}}{\sum_j s_j^{-\kappa}} = -1$, it suffices to show that $\zeta_j \leq 1$. It is easy to see that for $\kappa < \frac{\delta}{\lambda}$, which in turn implies

$$1 - \theta - \beta > (1-\lambda)\,\beta\left(1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}\right).$$

I emphasize that these are sufficient conditions, and the derivative should be well-behaved for a much broader set of parameter values.

In a symmetric equilibrium, the expression simplifies to

$$\begin{aligned} d\log s_k &= \frac{h_{\mathrm{F}}}{h_{\mathrm{D}} + h_{\mathrm{F}}\frac{\delta}{\lambda}}\left(\frac{N-1}{N}\right)d\log\tilde{\gamma}_k \\ &> 0 \end{aligned}$$

where I used the fact that $\chi = \frac{1}{N}$ by symmetry. For any other country, I have

$$\begin{aligned} d\log s_j &= -\frac{h_{\mathrm{F}}}{h_{\mathrm{D}} + h_{\mathrm{F}}\frac{\delta}{\lambda}}\frac{1}{N}d\log\tilde{\gamma}_k \\ &< 0. \end{aligned}$$

This means that a small increase of a country's research productivity around a symmetric steady state leads to an increase in the skill premium, while the skill premium falls in every other country. This contrasts with the autarky equilibrium where the skill premium is unrelated to research productivity.

[89] By the rank-nullity theorem, zero is an eigenvalue with multiplicity n-1, and the only eigenvalue left determining whether the system converges is given by the scalar product of the two vectors.

A.6 Hopenhayn Version

A.6.1 Static Problem

I solve the static problem of the firm first. I ignore the overhead adoption cost for now but it will show up in the dynamic problem, of course. Recall the production function

$$y_i = \left(\left(\int_{j \in \Omega_{A_i}} \left(\frac{x_j}{\alpha} \right)^\alpha \right) \left(\frac{l_i^\zeta}{\zeta (1-\alpha)} \right)^{1-\alpha} \right)$$

where the reader should take account of the curvature induced by $\zeta < 1$. Note that $\left(\frac{1}{\alpha}\right)^\alpha \left(\frac{1}{\zeta(1-\alpha)}\right)^{1-\alpha}$ is a normalizing constant for beautification purposes only. [90]

The firm solves the static problem –abstracting away from the technology adoption margin for now– where the price of the final good is normalized to unity, $P_y := 1$.

$$\max_{x_j, l_i} y_i - wl_i + \int p_{x,j} x_j dj \tag{70}$$

I solve this problem in two stages. First, consider a given expenditure on intermediate goods $p_x \int x_j dj := R_X$ and solve for the optimal demand for each intermediate good

$$x_j^D = \frac{R_X}{P_X^{\frac{\alpha}{\alpha-1}}} p_{x,j}^{\frac{1}{\alpha-1}}$$

where the ideal price index reads $P_X = \left(\int p_{x,j}^{-\frac{\alpha}{1-\alpha}} dj \right)^{-\frac{1-\alpha}{\alpha}}$. Given symmetry, all prices are identical so the firms optimally spreads total capital expenditure evenly across intermediates

$$x_j = \frac{X}{A_i} \; \forall j.$$

Now I plug this solution back into (70), and solve the following static maximization problem

$$\max_{X_i, l_i} \left(\left(\frac{X_i}{\alpha} \right)^\alpha \left(A_i \left(\frac{l_i}{\zeta (1-\alpha)} \right)^\zeta \right)^{1-\alpha} \right) - wl_i - p_x X_i$$

[90] One could easily put the curvature on the outer brackets as well without changing the economics of the paper, i.e., $y_i = \left(\frac{1}{\zeta} \left(\int_{j \in \Omega_{A_i}} \left(\frac{x_{ij}}{\alpha} \right)^\alpha dj \right) \left(\frac{l_i}{1-\alpha} \right)^{1-\alpha} \right)^\zeta$. This formulation, however, leads to slightly less elegant solutions.

which leads to the following first-order conditions

$$
\begin{aligned}
y_i \alpha &= X_i p_x \\
y_i \zeta (1-\alpha) &= l_i w.
\end{aligned}
$$

Factor payments are constant shares of revenue, due to the Cobb-Douglas assumption, so firm operating profits equal $\pi^o = y - l_i w - X_i p_x = (1-\zeta)(1-\alpha) y$. Moreover, after inverting the first order conditions and plugging them back into the production function, output appears as a function of of input prices, and importantly, of the productivity of the firm A_i

$$
\begin{aligned}
y_i &= \left(\frac{X_i}{\alpha}\right)^{\alpha} \left(A_i \left(\frac{l_i}{\zeta(1-\alpha)}\right)^{\zeta}\right)^{1-\alpha} \\
&= \left(\frac{y_i \alpha}{p_x \alpha}\right)^{\alpha} \left(A_i \left(\frac{\left(\frac{y_i \zeta (1-\alpha)}{w}\right)}{\zeta(1-\alpha)}\right)^{\zeta}\right)^{1-\alpha} \\
&= \left(\frac{y_i}{p_x}\right)^{\alpha} \left(A_i \left(\frac{y_i}{w}\right)^{\zeta}\right)^{1-\alpha} \\
&= y_i^{\alpha+\zeta(1-\alpha)} (p_x)^{-\alpha} \left(\frac{w^{\zeta}}{A_i}\right)^{-(1-\alpha)} \\
&= y_i^{\alpha+\zeta(1-\alpha)} (p_x)^{-\alpha} \left(\frac{w^{\zeta}}{A_i}\right)^{-(1-\alpha)} \\
&\Leftrightarrow \\
y_i &= \left\{ (p_x)^{-\alpha} \left(\frac{w^{\zeta}}{A_i}\right)^{-(1-\alpha)} \right\}^{\frac{1}{(1-\alpha)(1-\zeta)}}.
\end{aligned}
$$

Take account of the partial derivative of output with respect to productivity

$$
\frac{\partial \log y}{\partial \log A} = \frac{1}{1-\zeta}, \tag{71}
$$

which will become relevant for the dynamic problem considered next. Since $\pi = (1-\zeta)(1-\alpha) y$, it follows that $\frac{\partial \pi}{\partial A} = (1-\alpha) \frac{y}{A} = \frac{1}{1-\zeta} \frac{\pi}{A}$.

A.6.2 Dynamic Adoption Problem

I first solve the problem of an incumbent firm, i.e., the technology adoption problem, before I turn attention to the free entry condition. The dynamic problem reads

$$(r+\delta_X)\,V = \pi^o + \dot{V} + V_A\dot{A} - w_H h$$

s.t.

$$\dot{A} = \nu A^\theta A_F^{1-\theta} h^\beta - \delta_I A.$$

To make the problem stationary, I normalize the value function by the real wage of production labor, and I focus on the state variable $z := \frac{A}{A_F}$, which will be constant in equilibrium while A and A_F will be continuously growing. Define $v := \frac{V}{w}$, and $z := \frac{A}{A_F}$, and note that this change of variable has no effect on the underlying economics, which can now be studied in the stationary system

$$(r - g_w + \delta_X)\,v = \frac{\pi^o}{w} + \dot{v} + v_z\dot{z} - sh \tag{72}$$

s.t.

$$\dot{z} = \nu z^\theta h^\beta - (\delta_I + g_F)\,z.$$

An interior solution to the system (72) satisfies the first-order condition

$$h = \left\{\frac{v_z\beta\nu z^\theta}{s}\right\}^{\frac{1}{1-\beta}}, \tag{73}$$

which captures the trade-off between the cost of adoption in terms of the price of skilled labor, and the benefit of improving the firm's productivity.

To derive the differential equation that governs optimal skilled labor investment choices, I differentiate (72) with respect to z, and after using the Envelope theorem, I arrive at

$$(r - g_w + \delta_X) = \frac{\pi_z^o}{wv_z} + \frac{\dot{v}_z}{v_z} + \frac{v_{zz}\dot{z}}{v_z} + \nu\theta z^{\theta-1}h^\beta - (\delta_I + g_F)\,. \tag{74}$$

Now taking logs of (73) and differentiating with respect to time yields

$$\frac{\dot{h}}{h} = \frac{1}{1-\beta}\left\{\frac{\dot{v}_z}{v_z} + \frac{v_{zz}}{v_z}\dot{z} + \theta\frac{\dot{z}}{z} - \frac{\dot{s}}{s}\right\}. \tag{75}$$

Now plug (74) into (75) to get

$$\frac{\dot{h}}{h} = \frac{1}{1-\beta}\left\{(r - g_w + \delta_X) - \frac{\pi_z^o}{wv_z} - \theta\nu z^{\theta-1}h^{\beta} + (\delta_I + g_F) + \theta\frac{\dot{z}}{z} - \frac{\dot{s}}{s}\right\}$$
$$\frac{\dot{h}}{h} = \frac{1}{1-\beta}\left\{(r - g_w + \delta_X) - \frac{\pi_z^o}{wv_z} - \theta\nu z^{\theta-1}h^{\beta} + (\delta_I + g_F) + \theta\left[\nu z^{\theta-1}h^{\beta} - (\delta_A + g_F)\right] - \frac{\dot{s}}{s}\right\}$$
$$\frac{\dot{h}}{h} = \frac{1}{1-\beta}\left\{r - g_w + \delta_X + (1-\theta)(\delta_I + g_F) - \frac{\dot{s}}{s} - \frac{\pi_z^o}{wv_z}\right\}$$
$$\frac{\dot{h}}{h} = \frac{1}{1-\beta}\left\{r - g_w + \delta_X + (1-\theta)(\delta_I + g_F) - \frac{\dot{s}}{s} - \frac{\beta h^{\beta-1}\nu z^{\theta-1}}{s}\frac{\pi_z^o z}{\pi^o}\frac{\pi^o}{w}\right\}.$$

The final piece to derive the dynamic investment equation involves deriving the partial elasticity of profits with respect to productivity, which is $\frac{1}{1-\zeta}$ as can be seen in equation (71), which yields

$$\frac{\dot{h}}{h} = \frac{1}{1-\beta}\left\{r - g_w + \delta_X + (1-\theta)(\delta_I + g_F) - \frac{\dot{s}}{s} - \frac{\beta\nu z^{\theta}h^{\beta-1}}{s}\left[\frac{1}{1-\zeta}\frac{\pi^o}{w}\frac{1}{z}\right]\right\}. \tag{76}$$

For now, assume that a well-defined steady state exists, which I will prove later, together with stability and uniqueness properties, and assume that in such a steady state the adoption gap is constant and firms higher a constant number of skilled workers to adopt technology, i.e., $\dot{z} = 0$ and $\dot{h} = 0$ so that

$$z_* = \left(\frac{\nu}{\delta_I + g_F}\right)^{\frac{1}{1-\theta}}(h_*)^{\frac{\beta}{1-\theta}} \quad \text{(from resource constraint } \dot{z}\text{)}$$

Using this link, one can further simplify (76) to derive h_* in the steady state, which is inversely related to the skill premium,

$$r - g_w + \delta_X + (1-\theta)(\delta_I + g_F) = \frac{(\delta_I + g_F)}{h^* s}\frac{\beta}{1-\zeta}\frac{\pi}{w}$$
$$\Leftrightarrow$$
$$h_* = \frac{1}{s}\frac{(\delta_I + g_F)\frac{\beta}{1-\zeta}}{r - g_w + \delta_X + (1-\theta)(\delta_I + g_F)}\frac{\pi}{w}. \tag{77}$$

One can verify that the following inequality is necessary for a well-defined balanced growth path

$$\frac{\beta}{1-\zeta} < \frac{r - g_w + \delta_X}{\delta_I + g_F} + (1-\theta). \tag{78}$$

Proof. By contradiction, suppose that $\frac{\beta}{1-\zeta} \geq \frac{r-g_w+\delta_X}{\delta_I+g_F} + (1-\theta)$. In that case, spending on learning in the steady state, $h_* s$, is weakly larger than total operating profits $\frac{\pi^o}{w}$, so that the net profits $\frac{\pi}{w}$ =

$\left(\frac{\pi^{o}}{w}-sh_{*}\right)$ are non-positive. However, firms need to make positive profits to cover the initial fixed cost of entry. Thus, any well-defined balanced growth path requires (78) to hold. □

A.6.3 Free Entry

To close the model, Hopenhayn (1992) introduces a free entry condition such that the cost of entry, paid in labor, equals the present discounted value of the firm

$$f_{\mathrm{E}}w_{L,t}=\int VdF_{t}\left(A|\mathrm{E}\right). \tag{79}$$

In the main part of the paper I assume that $F\left(A|\mathrm{E}\right)=F\left(A\right)$ where $F\left(A\right)$ refers to the incumbent productivity distribution (expressing this in terms of total productivity A or relative productivity z is inconsequential since the firm takes the frontier A_{F} as given). The remaining steps are extremely similar to the baseline model.

A.7 Heterogenous Firms with Partial Knowledge Spillovers

A simplifying assumption in the paper is the complete knowledge spillover from incumbents to entrants. An alternative specification is one where the entrant only obtains a fraction $\lambda_{\mathrm{E}}\mathbb{E}\left[z\right]$ where $\lambda_{\mathrm{E}}<1$. This tweak turns the setting into a heterogeneous firm model where entrants learns from the most sophisticated incumbent, but imperfectly so. A well-defined equilibrium is characterized by a distribution $f\left(z\right)$ with support $z\in\left[\lambda_{\mathrm{E}}\mathbb{E}\left[z\right],z_{\max}\right]$. This leads to a normalized free entry condition

$$f_{\mathrm{E}}=v\left(\lambda_{\mathrm{E}}\mathbb{E}\left[z\right]\right).$$

Building on Hopenhayn (1992) and Melitz (2003), the profit ratio of any two firms can be expressed as $\frac{\pi_i}{\pi_j}=\left(\frac{z_i}{z_j}\right)^{(1-\alpha)(\sigma-1)}$, and normalized profits for firm i are given by $\frac{\pi(z_i)}{w}=\frac{(z_i)^{(1-\alpha)(\sigma-1)}}{\mathbb{E}\left[z^{(1-\alpha)(\sigma-1)}\right]}\frac{l_P}{m(\sigma-1)(1-\alpha)}$. Instead of using i subscript, I now index firms by z.

Next, consider the problem of some firm with productivity z in the steady state (so $\dot{v}=0$)

$$\left(\rho+\delta_{\mathrm{X}}\right)v\left(z\right)=\max_{h}\pi\left(z\right)-sh+\left(\partial_{z}v\right)\cdot\left[\nu z^{\theta}h^{\beta}-\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)z\right].$$

The first order condition still reads

$$h\left(z\right)=\left\{\frac{\left(\partial_{z}v\right)\beta\nu z^{\theta}}{s}\right\}^{\frac{1}{1-\beta}}.$$

I assume for simplicity that $\dot{s}=0$ and derive a similar dynamic investment equation as for the homogeneous firm case

$$\frac{\dot{h}_i}{h_i}(1-\beta) = (\rho + \delta_{\mathrm{X}} + (1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})) - \frac{\beta \nu z^{\theta} h^{\beta-1}}{s}\left[\frac{\pi}{w}\frac{(1-\alpha)(\sigma-1)}{z}\right].$$

It is useful to rewrite this expression relative to the firms with the maximum productivity

$$\begin{aligned}\frac{\dot{h}_i}{h_i}(1-\beta) &= (\rho + \delta_{\mathrm{X}} + (1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})) \\ &\quad - \frac{\beta(1-\alpha)(\sigma-1)(z_{\max})^{\theta-1} h_{\max}^{\beta-1}\nu}{s}\frac{\pi_{\max}}{w}\left(\frac{z_i}{z_{\max}}\right)^{\theta-1+(1-\alpha)(\sigma-1)}\left(\frac{h_{\max}}{h_i}\right)^{1-\beta}\end{aligned}$$

which helps to pin down the equilibrium dynamics. By construction, the most productive firm hires a constant amount of skilled labor with the only difference to the homogenous firm model being that the steady state profits are larger. This is a direct consequence of starting out with an initially lower productivity. Higher long-run profits have to make up for low profits after the firm just entered, since the entry cost are the same in both cases.

For this equilibrium to be well-defined, I need it to be true that entering firms improve their productivity so that they converge to the more profitable incumbents in the long-run. This is not trivial. One way to see this is to sue a phase-diagram in the h-z space assuming a stationary equilibrium exist for some $z_{\max}$. Since there is no risk the differential equation I have derived beforehand applies. Slightly rewriting and ignoring a a scalar $1-\beta$ (which does not matter for the issue of existence but surely matters for the speed of convergence) leads to

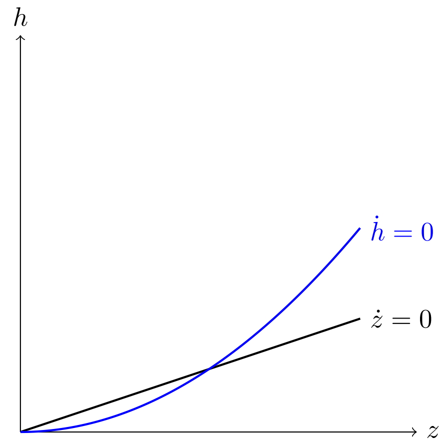
$$\frac{\dot{h}}{h} = \kappa_1\left(1 - \left(\frac{z}{z_{\max}}\right)^{\theta-1+(1-\alpha)(\sigma-1)}\left(\frac{h_{\max}}{h}\right)^{1-\beta}\right).$$

Assuming $\dot{h}=0$ implies $h = \left(\frac{z}{z_{\max}}\right)^{\frac{\theta-1+(1-\alpha)(\sigma-1)}{1-\beta}} h_{\max}$. Moreover, assume $\frac{\theta-1+(1-\alpha)(\sigma-1)}{1-\beta} > 0$ so the $\dot{h}$ locus is upward-sloping in z. Moreover, note that the $\dot{z}$ locus implies a positive link between h and z as well, $h = \left(\frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{\nu} z^{1-\theta}\right)^{\frac{1}{\beta}}$. When

$$\frac{\theta-1+(1-\alpha)(\sigma-1)}{1-\beta} < \frac{1-\theta}{\beta},$$

one can show that the z-locus cuts the h-locus once from below. The associated stability analysis suggests convergence from below, see figure 13.

Figure 13. Phase Diagram



This structure gives rise to a meaningful stationary distribution whereby firms start out small and improve their productivity over time. Several features are noteworthy. First, the firm size distribution is independent of the relative price of skill s. What this suggests is that a new stationary equilibrium with a higher price of skill produces an identical wave but shifted to the left, i.e., a permanently lower level of adoption across all firms. This traveling wave property is not surprising in light of recent work on heterogeneous firms, see Luttmer (2007), König, Lorenz, and Zilibotti (2016), Sampson (2016), Benhabib, Perla, and Tonetti (2021) and Perla and Tonetti (2014). Crucially, the partial equilibrium elasticity $\frac{\partial \log \mathbb{E}[z]}{\partial \log s} = -\frac{\beta}{1-\theta}$ computed in the main text still applies. Second, demand for skilled labor in the production sector can be derived by integrating over all productivity levels

$$h_{\mathrm{D}} = m \int_{\lambda_{\mathrm{E}} \mathbb{E}[z]}^{z_{\max}} f(z)\, h(z)\, dz.$$

Third, the innovator problem in the steady state needs to be updated as follows

$$V = \mathbb{E}_z \left[V(z) \right] \tag{80}$$

where $V(z) = \int_{t+\tau(z)}^{\infty} \exp\left(-\int_t^s \left[r_u + \delta_{\mathrm{X}}\right] du\right) \pi_{\mathrm{I}}(z, s)\, ds$ is a function of the firm-specific z-level which matters both in terms of firm size and how long it takes for an idea to be adopted by a firm of type z. The problem is conceptually the same as before except now one needs to keep track of the distribution of firm-specific adoption gaps. Of course, equation (80) is also conceptually very close to the value function of an innovator in the open economy in the main text, which is a discretized version of (80) where z's pertain to different countries.

A.8 Other Model Extensions

Stochastic Adoption. Since asset markets are complete and there are no stochastic shocks, risk plays no role when potential innovators consider entry into innovation. It is thus not surprising that stochastic adoption does not change any of the results qualitatively.

For example, a different version that I have experimented with is to let un-adopted ideas to be uniformly sampled at Poisson rate $\frac{A(g_A+\delta_I)dt}{A_F-A} = \frac{z}{1-z}(g_A+\delta_I)$ where $\frac{1}{A_F-A}$ is the uniform density and $A(g_A+\delta_I)\,dt$ is the flow of new ideas that are adopted at each instant. The probability density is then simply the product of the two, given statistical independence. On a balanced growth path with constant relative technology level z, it is again true that a z close to unity makes the adoption friction vanish. In contrast, as z approaches zero, the net present value of an innovation falls to zero as well since the adoption probability converges to zero as well.

Using this alternative functional form, one can follow the same steps as in the main text and compute the expected present discounted value of a patent. The insight that adoption and innovation are complementary on the market for ideas are robust to this alternative functional form. While stochastic adoption is more realistic in the sense that most innovators do not know when, if ever, their idea becomes profitable, this version of the model would be slightly less tractable regarding the market clearing condition for skilled labor.

The value function of an innovator would now look as follows

$$(r-g_V+\delta_{\mathrm{I}})\,V_{\mathrm{I}} = 0+\lambda(z)\left(V_{\mathrm{I}}^{\text{adopted}}-V_{\mathrm{I}}\right) \tag{81}$$

where $g_V = \frac{\dot{V}}{V}$ is pinned down by the free entry condition, $\lambda(z) = \frac{z}{1-z}(g_A+\delta_I)$, and $V_{\mathrm{I}}^{\text{adopted}} = \int_t^\infty e^{-\int_t^u r_v+\delta_{\mathrm{I}}dv}\pi_{\mathrm{I}}(u)\,du$. The zero in (81) is meant to highlight that there are no flow profits up until the idea is adopted.

Production Labor in Innovation and Upward-Sloping Skilled Labor Supply. Another natural extension is to allow production labor to be used in the research sector as well, i.e., suppose that the entry cost combine each labor type according to a Cobb-Douglas production function with skilled labor share κ, so the entry cost becomes $f_{\mathrm{R}}\left(\frac{w_H}{\kappa}\right)^\kappa\left(\frac{w}{1-\kappa}\right)^{1-\kappa}$. In that case, an expansion of the research sector will simultaneously raise demand for production labor.[91] It will still be true, though, that the impact on production worker wages can be characterized by the evolution of frontier technology A_{F} and the relative technology level z alone. Instead of making additional assumptions on the factor intensity in each sector, one can capture these considerations by allowing the supply of skilled labor to be upward sloping in the skill premium. To this end, suppose that an exogenously growing popu-

[91] See Helpman (2016) for a summary of the large literature on factor-biased trade and inequality.

lation of workers split between production and skilled labor $N = L + H$ where $\psi(s)$ is the share of workers that are skilled, which is now an upward sloping function of the skill premium. The market clearing condition for skilled labor now reads

$$\frac{1}{s} z^{-\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} \Lambda_{\mathrm{F}} + \frac{1}{s} \Lambda_{\mathrm{D}} = \frac{\psi(s)}{1 - \psi(s)}.$$

I assume $\frac{\psi(s)}{1-\psi(s)} = \psi_0 (s - \kappa)^{\eta}$ where the baseline case refers to $\eta \to 0$. As $\eta \to \infty$ and skilled labor supply becomes perfectly elastic,[92] the skill premium is exogenously fixed at $s = \kappa > 1$. In that scenario, the adoption margin is mute, and the implications of the theory with respect to a market integration shock are extremely similar to the simpler model of Jones (1995) without adoption margin and one type of labor. I highlight the role of skill scarcity in driving weak technology adoption. Surely skilled labor supply is somewhat elastic, but not nearly enough to counteract the substantial increase in the skill premium observed over the past couple of decades.

Government & Complementary Infrastructure. A classic theme in the growth literature is government capacity, and the important role of complementary pubic infrastructure investment. For example, the adoption of motorized vehicles will not occur unless the government builds roads. An easy way to incorporate this aspect into the model is to generalize the adoption technology with a notion of public infrastructure $A_{\mathrm{G},t}$ so that the net flow of ideas reads $\dot{A}_i = A_{\mathrm{F}}^{1-\theta} A_{\mathrm{G}}^{(1-\phi)\theta} A_i^{\phi\theta} - \delta_{\mathrm{I}} A_i$. A normalized version simplifies to $\dot{z}_i = \nu z_{\mathrm{G}}^{(1-\phi)\theta} z_i^{\phi\theta} h_i^{\beta} - (\delta_{\mathrm{I}} + g_{\mathrm{F}}) z_i$ where the firm takes z_{G} as given. Without complementary government investment z_{G} will be zero and the returns to adoption are zero as well. If the government invests in $A_{\mathrm{G}}^{(1-\phi)\theta}$ at a rate consistent with a balanced growth path, then all of the previous derivations go through after updating the term θ by a factor of ϕ. Note that capable government employees are likely to be instrumental in building up complementary public infrastructure. The role of skilled labor remains pivotal.

Skill Biased Technological Change. A common explanation for rising inequality is based on theories of skill-biased technological change, see Katz and Murphy (1992). Goldin and Katz (2010) present compelling empirical evidence from a number of studies covering almost two centuries that show how skill-biased technological change has shaped labor market outcomes. It is thus useful to consider how my model relates to this large literature. I generalize the model to include allow for changing task-content of work by modeling intermediate goods production as $y = \left((Ax)^{\alpha} l^{1-\alpha}\right)^{1-\tilde{\beta}} h^{\tilde{\beta}}$ so that both production and skilled labor enters the production function ($\tilde{\beta} = 0$ is the baseline case in the paper).[93] The model remains mostly unchanged except for an additional term $\tilde{\Lambda}_{\tilde{\beta}}$ in the labor market clearing condition,

$$\frac{1}{s} \left(\tilde{\Lambda}_F z^{\frac{\tilde{\rho}}{g_F + \delta_I}} + \tilde{\Lambda}_D + \tilde{\Lambda}_{\tilde{\beta}} \right) = h_{tot}. \tag{82}$$

[92] I do not model the actual investment cost, see for instance Acemoglu (2009) for a model with explicit schooling choice.

[93] Acemoglu and Restrepo (2020) show how to micro-found this Cobb-Douglas production function in a model of automation.

A changing task content is captured in an increase in $\tilde{\beta}$ (or $\Lambda_{\tilde{\beta}}$) and would raise the overall price of skill. This would push down aggregate growth as less skilled labor is available for innovation and adoption. As production requires more skill, less is available to invest in innovation and adoption.

As pointed out in Acemoglu and Autor (2011), skill-biased technological change generates wage growth *for all workers.* The reason is the strong complementarity between high and low skilled workers which ensures that technological change benefits everyone, even if it is biased. The theory proposed here is complementary to this literature by pointing out that a reallocation of skill across space or sectors can create real wage losses whenever skill is an important input to technology adoption. If so, the skill premium takes on a new role where an increase in the relative price of skilled labor reduces equilibrium adoption effort and thus hampers economic growth.

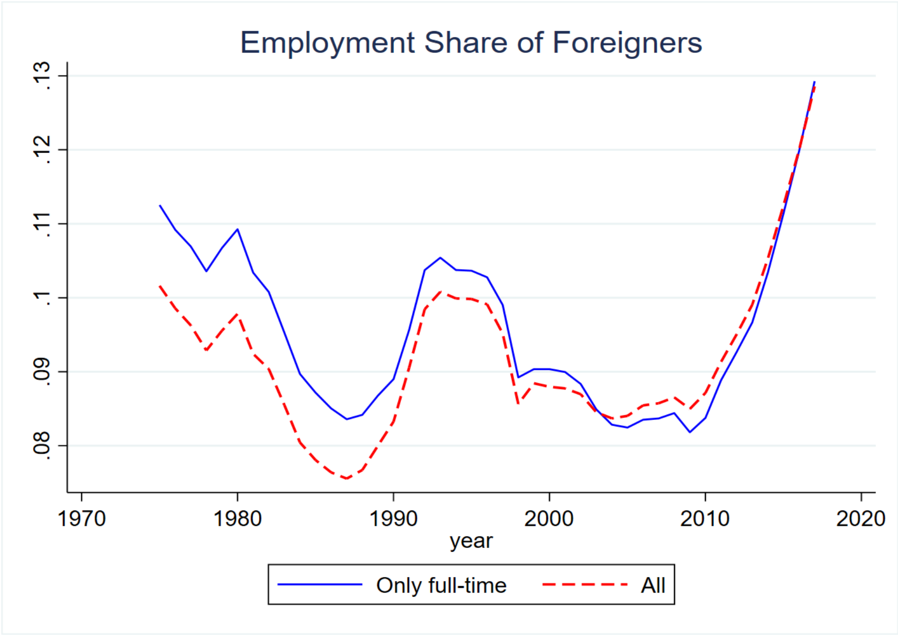
Note, however, that an increase in the relative price of skill driven by a changing task content of work will hit the innovation sector the hardest due to the second round effects through a rising adoption gap as $z^{\frac{\tilde{\beta}}{g_F+\delta_I}}$ falls. A changing task content of work is thus consistent with sluggish growth and rising inequality in this model, but it will not allow innovative activity to take off. The effect of globalization on the returns to innovation will resolve this tension and help make sense of rising innovative activity in advanced economies.

Immigration. A fully integrated equilibrium with free migration behaves differently from the baseline model. Unskilled migration in the baseline model reduces production worker wages in the advanced economy in two ways. First, a direct negative effect on the unskilled wage as their factor becomes relatively more abundant. Second, since there are more production workers, more skilled labor needs to be devoted to technology adoption, further hurting innovation and reducing frontier innovation.

Skilled labor inflows, on the other hand, are extremely beneficial for unskilled workers as they lead to both heightened adoption and innovation. Interestingly, the effect on skilled labor is ambiguous and likely positive as well. Even though their relative wage has to fall, greater adoption and rising innovation can lift up the high skilled workers' wages.

Figure 14 helps us assess the relevance of immigration into Germany as a potential reason for weak wage growth. It turns out that the foreign employment share is falling since the mid 1990s, leading to an all-time low in the 2000s. This figure suggests that trends in immigration are unlikely to account for wage stagnation in Germany at the turn of the century. This is not to say that immigration doesn't matter – it seems hard to think about technological frontier growth, especially in the US, without the input of foreign researchers and entrepreneurs. The model can in principal be used to explore this issue, and consider the effect of brain drain and innovation gains in a global integrated equilibrium with realistic scale effects. I do not want to take up this task in this paper.

Figure 14. Immigration



The figure plots the share of foreign workers in West Germany, using the BHP of the IAB.

B Computational Appendix

Here I show how to compute the solution and solve for transition dynamics based on the finite difference method in Achdou et al. (2022).

B.1 Closed Economy

B.1.1 Production Sector Firms' Problem

The normalized firm problem reads

$$(r_t + \delta_{\mathrm{X}} - g_w)\, v(z, X) = \frac{\pi(z)}{w} - sh + \partial_z v \cdot \left[\nu z^{\theta} h^{\beta} - (\delta_{\mathrm{I}} + g_{\mathrm{F}})\, z\right] + \mathcal{A} v$$

subject to the law of motion for z_i and the evolution of aggregate state variables captured in X, including the equilibrium measure of firms, among other things. The framework can be readily extended to include stochastic productivity shocks in $\mathcal{A}$, but I drop this term here.

The first-order condition reads

$$\left\{\frac{(\partial_z v)\, \beta \nu z^{\theta}}{s}\right\}^{\frac{1}{1-\beta}} = h,$$

and I use the following boundary conditions

$$\left(\partial_z v^{\mathrm{f}}\right) = \frac{s}{\beta \nu \overline{z}^{\theta}} \left\{\frac{(\delta_{\mathrm{I}} + g_{\mathrm{F}})\, \overline{z}^{1-\theta}}{\nu}\right\}^{\frac{1-\beta}{\beta}}$$
$$\left(\partial_z v^{\mathrm{b}}\right) = \frac{s}{\beta \nu \underline{z}^{\theta}} \left\{\frac{(\delta_{\mathrm{I}} + g_{\mathrm{F}})\, \underline{z}^{1-\theta}}{\nu}\right\}^{\frac{1-\beta}{\beta}}$$

for forward and backward difference, respectively, where $\overline{z}$ and $\underline{z}$ represent the highest and lowest value of z over the grid space.

Rewrite the problem as follows

$$(r_t + \delta_{\mathrm{X}} - g_w)\, v(z) = B_t z^{(1-\alpha)(\sigma-1)} - sh + \partial_z v \cdot \left[\nu z^{\theta} h^{\beta} - (\delta_{\mathrm{I}} + g_{\mathrm{F}})\, z\right] \tag{83}$$

and note that B_t captures, among other things, the equilibrium measure of firms as well as the properly weighted average productivity level $\tilde{z} = \left\{\mathbb{E}\left[z^{(1-\alpha)(\sigma-1)}\right]\right\}^{\frac{1}{(1-\alpha)(\sigma-1)}}$, where the heterogeneity arises only for the case of imperfect spillover at entry, $\lambda_{\mathrm{E}} < 1$.

Note that $B_t M \tilde{z}^{(1-\alpha)(\sigma-1)} = \frac{Y}{\sigma}$, which can be rearranged to

$$B_t = \frac{l_{\mathrm{P}}}{m\tilde{z}^{(1-\alpha)(\sigma-1)}} \frac{1}{(1-\alpha)(\sigma-1)}.$$

Moreover, in the steady state $m = \frac{1-l_{\mathrm{P}}}{(g_L+\delta_{\mathrm{X}})f_{\mathrm{E}}}$. If I had a value for B_t, and a solution to (83), I could infer all endogenous variables. To find B_t, I solve (83) for a guess of B_t and check if the free entry condition holds

$$f_{\mathrm{E}} = \mathbb{E}_{z_{\mathrm{E}}}\left[v\left(z_{\mathrm{E}}\right)\right].$$

I raise the value of B_t if the cost of entry is higher than the present discounted value using a bisection method.

The density of incumbents g_z and entrants $g_{z_{\mathrm{E}}}$ follows from the KFE equation

$$\dot{g}_z = \mathcal{A}^T\left(g_z\right) g_z$$

where I make clear that the infinitesimal generator $\mathcal{A}^T\left(g_z\right)$ is itself a function of the stationary density since entrants' productivity is assumed to be a function of the average $z_{\mathrm{E}} = \lambda_{\mathrm{E}}\mathbb{E}\left[z\right]$. Note that a flow of $g_L + \delta_{\mathrm{X}}$ firms enters at productivity z_{E}, accounting for both net entry rate (which is the same as long run population growth) and exogenous firm death. In matrix form, this means I have to add $-\left(g_L+\delta_{\mathrm{X}}\right)$ on the diagonal, and $\left(g_L+\delta_{\mathrm{X}}\right)\mathbb{1}_{z=z_{\mathrm{E}}}$in each column of $\mathcal{A}^T$.[94]

Having solved for B_t and g_z, values for l_{P} and m follow as a function of the skill premium

$$\begin{aligned} l_{\mathrm{P}} &= \frac{B_t\tilde{z}^{(1-\alpha)(\sigma-1)}}{\frac{(g_L+\delta_{\mathrm{X}})f_{\mathrm{E}}}{(1-\alpha)(\sigma-1)} + B_t\tilde{z}^{(1-\alpha)(\sigma-1)}} \\ m &= \frac{1-l_{\mathrm{P}}}{\left(g_L+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}} \end{aligned}$$

which in turn imply aggregate demand for skilled labor from the production sector

$$h_{\mathrm{D}} = m \cdot \int h\left(z\right) g_z dz.$$

B.1.2 Research Sector Problem

An alternative approach uses the following normalization

$$v_{\mathrm{I}} := \frac{V_{\mathrm{I}}}{w_L A_{\mathrm{F}}^{-\phi}},$$

[94] If z_{E} falls between two grid points, I turn the entry draw into a probabilistic one according to how close z_{E} is to either grid point.

which leads to the following recursion

$$v_{\mathrm{I}}\left(r_t - g_{w_L} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F}}\right) = e^{-\int_t^{t+\tau}\left(r_t - g_{w_L} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F}}\right)dv} \frac{\alpha l_{\mathrm{P},t+\tau}}{z_{t+\tau} a_{\mathrm{F},t+\tau}} \cdot \left[1 + \dot{\tau}_t\right] + \dot{v}_{\mathrm{I}},$$

and in discretized form

$$\begin{aligned} v_{\mathrm{I},t}\left(r_{t+\Delta} - g_{w_L,t+\Delta} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F},t+\Delta}\right) = e^{-\int_t^{t+\tau}\left(r_t - g_{w_L} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F}}\right)dv} \frac{\alpha l_{\mathrm{P},t+\tau}}{z_{t+\tau} a_{\mathrm{F},t+\tau}} \cdot \left[1 + \dot{\tau}_t\right] \\ + \frac{v_{\mathrm{I},t\,+\,\Delta} - v_{\mathrm{I},t}}{\Delta}. \end{aligned}$$

For a free entry equilibrium, I want to make sure at all times that

$$v_{\mathrm{I}} \leq \frac{1}{\gamma} h_{\mathrm{F}}^{1-\lambda} s$$

holds. This recursive representation can be used to compute equilibrium in the research sector off the balanced growth path.

B.1.3 Transition Dynamics

Production sector. Taking the sequence $\{s_t, g_{\mathrm{F},t}, B_t\}_{t\in[0,T]}$ as well as initial and terminal conditions as given, I solve the production firms' problem along the transition path. The first order condition now reads

$$\left\{\frac{\left(\partial_z v_{n+1}\right)\beta\nu z_{n+1}^{\theta}}{s_{n+1}}\right\}^{\frac{1}{1-\beta}} = h,$$

where each n is a step in time. Given a solution, the value function in v_n follows recursively.

Firm entry in production. In principal, the entry margin responds to technology adoption along the transition path. In particular, when the knowledge spillover at entry is strong high technology adoption effort coincides with lower firm entry so as to respect the free entry condition.

Given a solution for $\{v_{\mathrm{E}}\}$, I can asses whether the value of entry equals the entry cost f_{E}, and update labor devoted to firm entry accordingly. Computationally, this is inconvenient because the linear entry technology leads to bang-bang solutions, which are avoided in general equilibrium but cumbersome to solve for during the transition.

I simplify the model by assuming that $l_{\mathrm{P},t} = l_{\mathrm{P,ss}}$, i.e., a constant amount of labor is devoted to entry consistent with the long run steady state level, which itself is only a function of constant exogenous parameters. We will see that this simplifying assumption, which also makes capital accumulation easier since the effective number of production workers won't jump around during the transition, is quantitatively innocuous in that the value of entry into the production sector deviates less than 1% from its long-run steady state.

Technology Adoption and Waiting time. The model is hard, in part, because innovators need to keep track of changes in τ, and multiple τ's in the open economy version. Note that τ_0 is a jump variable that is implicitly defined by

$$\tau_0 = -\frac{\log z_0}{\frac{\int_0^{\tau_0} g_A(x)+\delta_{\text{I}} dx}{\tau_0}},$$

and for a given evolution of $A(t)$ one can use a simple bisection to get τ_0. Once the initial wait time is determined, one can use the law of motion of the waiting time

$$\dot{\tau}_t = \frac{g_{\text{F},t} - g_{A,t+\tau}}{\delta_{\text{I}} + g_{A,t+\tau}},$$

which can be inverted, and using the discrete time step approximation, yields next periods wait time as a function of frontier growth, productivity growth, and the previous wait time

$$\tau_{t+\Delta} = \tau_t + \Delta \cdot \frac{g_{\text{F},t} - g_{A,t+\tau}}{\delta_{\text{I}} + g_{A,t+\tau}}.$$

Research Sector. Note that for a guess $\{s_t\}$, and adoption choices in the production sector, the recursion derived beforehand implies a sequence $\{v_{\text{I}}\}$. Together with the entry technology into innovation, this sequence implies a sequence of skilled labor devoted to innovation so as to ensure that free entry holds.

General Equilibrium. Key inputs in solving the research and production sector's forward-looking problems are wage growth and interest rate. Note that wages equal

$$w_{L,t} = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \frac{\sigma-1}{\sigma} A_t k_t^{\alpha}$$

where $k_t = \frac{K_t}{A_t L_{\text{P},t}}$ is effective capital per production worker. Wage growth equals

$$g_{w_L} = g_{\text{F}} + g_z + \alpha g_k,$$

where g_z and g_k are zero in the steady state.

Similarly, the interest rate equals

$$r_t = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{\sigma-1}{\sigma} \alpha k_t^{\alpha-1} - \delta_k.$$

To proceed, I need to pin down the evolution of the normalized capital stock k_t, which follows from the capitalists' problem.

Simplified Physical Capital Accumulation. I simplify by assuming that a constant fraction χ_s of domestic final output is reinvested in physical capital. I pick χ_s such that the autarky steady state is consistent with the solution to the representative household problem in the steady state in Autarky. Given this assumption, the law of motion of physical normalized capital $k = \frac{K}{AL_{\mathrm{P}}}$ reads

$$\frac{\dot{k}}{k} = \chi_s \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} k^{\alpha-1} - (g_L + g_{l_{\mathrm{P}}} + g_{\mathrm{F}} + g_z + \delta_k).$$

In discretized form, I have

$$k_{t+\Delta} = k_t \cdot e^{\Delta\left[\chi_s \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} k^{\alpha-1} - \left(g_L + g_{l_{\mathrm{P}}} + g_{\mathrm{F}} + g_z + \delta_k\right)\right]}$$

where initial capital is given, and growth rates are computed as $\frac{\log\left(\frac{A_{t+\Delta}}{A_t}\right)}{\Delta}$. Based on the previous assumption, l_{E} is assumed to be constant. But note that even in the case of endogenous entry in the production sector, l_{E} only jumps at the very first moment, and is a smooth function of time for any $t > t_0$.This initial jump should be incorporated in the normalized starting value k_0. In the simplified case, k_0 is fully predetermined, and initial and long-run value coincide.

Given this law of motion of capital, together with optimal firm entry, the equilibrium interest rate follows

$$r_t = \left(\frac{\sigma-1}{\sigma}\alpha\right) \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \alpha k_t^{\alpha-1} - \delta_k$$

where the normalized capital stock in steady state reads

$$k_{\mathrm{ss}} = \left\{\frac{\chi_s}{g_L + g_{\mathrm{F}} + \delta_k}\right\}^{\frac{1}{1-\alpha}} \alpha^{-\frac{\alpha}{1-\alpha}} (1-\alpha)^{-1}.$$

When the saving rate, which is kept constant along the transition path, equals $\chi_s = \left(\frac{\sigma-1}{\sigma}\alpha\right) \cdot \alpha \frac{g_L + g_{\mathrm{F}} + \delta_k}{\rho + g_{\mathrm{F}} + \delta_k}$, I get $k_{\mathrm{ss}} = \left\{\left(\frac{\sigma-1}{\sigma}\alpha\right) \cdot \frac{\alpha}{\rho + g_{\mathrm{F}} + \delta_k}\right\}^{\frac{1}{1-\alpha}} \alpha^{-\frac{\alpha}{1-\alpha}} (1-\alpha)^{-1}$, which coincides with the normalized capital stock in the steady state when the saving rate is endogenous.

Algorithm. Note that I have simplified the problem so that the key endogenous object we are after is the sequence of skill premia $\{s_t\}$, which clear the market for skilled labor, and are consistent with forward-looking technology adoption and innovation choices in both production and research sector.

An algorithm that I have found to work well proceeds as follows.

1. Guess $\{s\}$.

2. Guess $\{a_{\mathrm{F}}\}$, solve optimal adoption choice backward, and $\{\tilde{z}_{t,}\}$, $\{r_t\}$ forward. Iterate until convergence. Compute $\{\tau\}$, which depends on $\{z\}$ and $\{a_{\mathrm{F}}\}$.

3. Solve innovator's problem holding $\{z, r\}$ fixed

(a) Holding τ fixed, solve the innovator problem backwards (there is not much to solve here other than computing the value of an idea recursively). Get a sequence of $\{v_{\mathrm{I}}\}$.

   i. Given a sequence of $\{s\}$ and $\{v_{\mathrm{I}}\}$, derive skilled labor devoted to entry $h_{\mathrm{F},t} = \left\{\frac{\gamma \cdot v_{\mathrm{I},t}}{s_t}\right\}^{\frac{1}{1-\lambda}}$

   ii. Use $\{h_{\mathrm{F}}\}$ to derive new sequence $\{a_{\mathrm{F}}^{\text{new}}\}$ using the resource constraint related to the creation of new ideas

   iii. Update $a_F^{\text{next}} = relax * \{a_{\mathrm{F}}\} + (1 - relax) * \{a_{\mathrm{F}}^{\text{new}}\}$

   iv. Go back to a) until convergence occurs, $a_{\mathrm{F}} \approx a_{\mathrm{F}}^{\text{new}}$.

(b) Update $\{\tau\}$ (I am still holding $\{z\}$ fixed but $\{\tau\}$ changes nonetheless because the frontier a_{F} is moving) and go back to a), stop when $\tau^{next} \approx \tau$, i.e., τ has converged to previous guess.

4. Go back to 2. using a new updated guess $\{a_{\mathrm{F}}\}$. Keep iterating from 2. – 4. until convergence.

5. Now you have two sequences $\{h_{\mathrm{D}}, h_{\mathrm{F}}\}$ that are consistent with optimization and resource constraints in each sector. The only thing that remains to be checked is whether the sequences are consistent with market clearing for skilled labor. Most likely they are not and so we compute excess demand functions and update the sequence of skill prices gently in the right direction

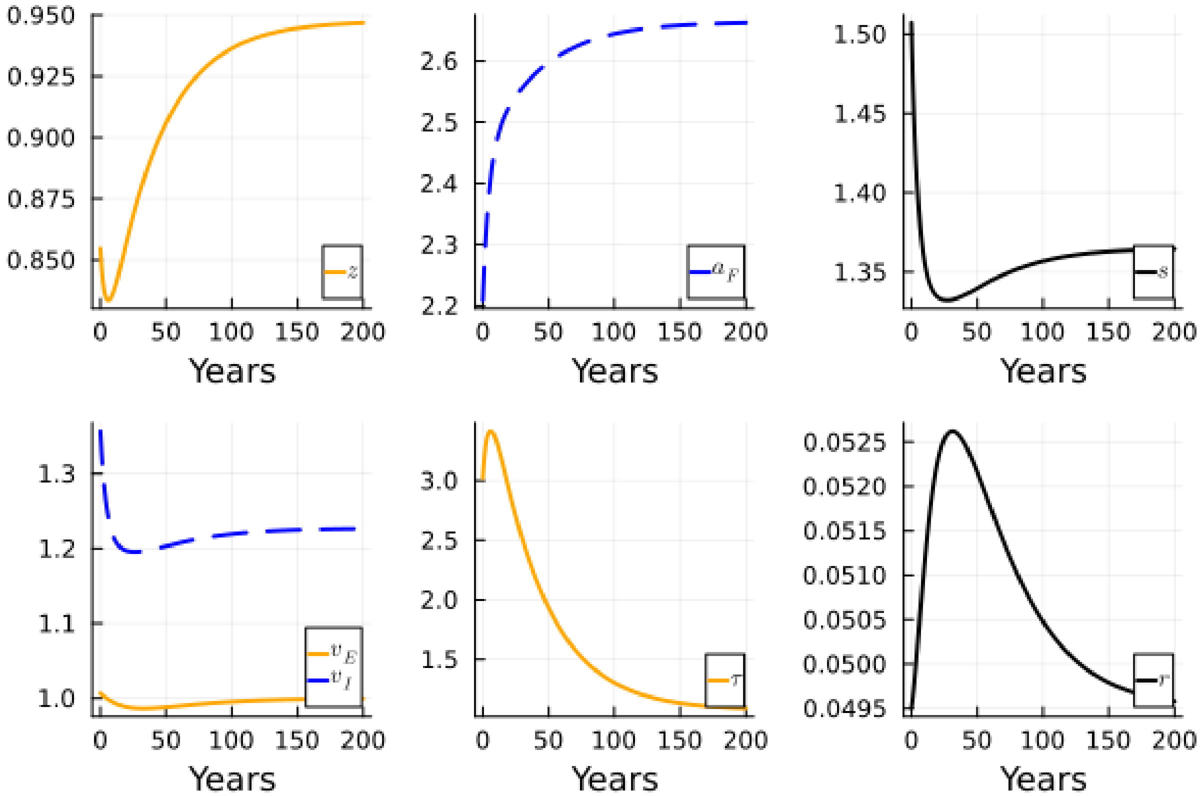
$$s^{\text{next}} = relax * s + (1 - relax) * s * e^{\theta_s[(h_{\mathrm{D}}+h_{\mathrm{F}})-h_{\text{tot}}]}$$

6. Go back to step 1. and start all over again for the new skill price guess up until convergence, i.e., $s^{next} \approx s$, which coincides with skilled labor market clearing $h_{\text{tot}} = h_{\mathrm{D}} + h_{\mathrm{F}}$

The figure below computes transition dynamics for a closed economy that is calibrated as in the main text. I shock the relative skilled labor supply from .15 to .18 by raising the skilled labor supply in the form of an unanticipated persistent shock at time zero. The transitions are then computed using the previous algorithm under perfect foresight. Figure 15 plots the results.

Three points are noteworthy. First, note that innovation responds quicker than technology adoption in the sense that convergence to its long-run steady state is faster. This is explained by the advantage of backwardness, which naturally leads to a lagged response of technology adoption. Second, the dynamics of the skill premium are non-monotone, and a fast initial fall in the price of skill is followed by slow but small rise in the price of skill later on. The increase in the price of skill, which ultimately is a function of the demand for skilled labor in this general equilibrium model, is directly related to increasing and lagged technology adoption effort. Third, the value of entry in the production sector is hardly moving, and deviates from its long-run value by less than 1% highlighting that the simplification is unlikely to matter quantitatively for the transition dynamics. The evolution of the value of entry into the research sector roughly follows the skill premium, and the difference in the two series is due to congestion whenever $\lambda < 1$.

Figure 15. Skilled Labor Expansion in Autarky



In this exercise I increase the relative skilled labor supply from .15 to .18. All of this increase occurs at time zero.

B.2 Open Economy

To solve for transition dynamics in the open economy, I follow similar steps. Note that the problem of production sector firms is unchanged. Firms in the research sector now need to take into account both opportunities abroad and competition at home when they consider whether to pay the fixed cost of producing an idea.

B.2.1 Innovation

The value of an innovation generalizes as follows in the open economy where I focus on the case of differences in size captured by country-specific production labor endowments L_c. I consider a version where I use L from the home economy as the normalizing factor so $b_k = \frac{L_k}{L}$ relative to the home economy. The value of an idea reads

$$
\begin{aligned}
V_{\mathrm{I}}\left(r+\delta_{\mathrm{I}}\right) &= \sum_{c} e^{-\int_{t}^{t+\tau_c}(r_v+\delta_{\mathrm{I}})dv}\pi_{\mathrm{I},c,t+\tau}\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{V}_{\mathrm{I}} \\
&= \sum_{c} e^{-\int_{t}^{t+\tau_c}(r_v+\delta_{\mathrm{I}})dv}\frac{\alpha L_{\mathrm{P},c,t+\tau_c}w_{L,c,t+\tau_c}}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{V}_{\mathrm{I}} \\
&= \sum_{c} e^{-\int_{t}^{t+\tau_c}(r_v+\delta_{\mathrm{I}})dv}\frac{\alpha l_{\mathrm{P}}w_{L,c,t+\tau_c}b_c}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}L\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{V}_{\mathrm{I}},
\end{aligned}
$$

where I used the fact that the allocation of production labor in production vis-a-vis entry is kept constant.

Next, use the normalization $v_{\mathrm{I},k,t} := \frac{V_{\mathrm{I}}}{w_{L,k}\left(A^{\mathrm{W}}_{\mathrm{F}}\right)^{-\phi}}$. Note that the normalization is country-specific because wages are. The normalized value of an idea reads

$$
\begin{aligned}
v_{\mathrm{I},k,t}\left(r_k-g_{w_{L,k}}+\delta_{\mathrm{I}}+\phi g_{\mathrm{F}}\right) &= \sum_{c}\frac{e^{-\int_{t}^{t+\tau_c}(r_k+\delta_{\mathrm{I}})dv}}{w_{L,k,t}}\frac{\left(A^{\mathrm{W}}_{\mathrm{F},t}\right)^{\phi}}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}\alpha L_{\mathrm{P},c,t+\tau_c}w_{L,c,t+\tau_c}\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{\tilde{v}}_{\mathrm{I}} \\
&= \sum_{c}\frac{e^{-\int_{t}^{t+\tau_c}(r_k+\delta_{\mathrm{I}})dv}}{w_{L,k,t}}\left(\frac{A^{\mathrm{W}}_{\mathrm{F},t}}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}}\right)^{\phi}\frac{L_{t+\tau_c}\alpha l_{\mathrm{P}}w_{L,c,t+\tau_c}}{\left(A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}\right)^{1-\phi}z_{c,t+\tau_c}}b_c\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{\tilde{v}}_{\mathrm{I}} \\
&= \sum_{c} e^{-\int_{t}^{t+\tau_c}\left(r_k-g_{w_L}+\delta_{\mathrm{I}}+\phi g_{\mathrm{F}}\right)dv}\left(\frac{\alpha l_{\mathrm{P}}}{a^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}\frac{w_{L,c,t+\tau_c}}{w_{L,k,t+\tau_c}}\right)\cdot b_c\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{\tilde{v}}_{\mathrm{I}}
\end{aligned}
$$

where the second line uses the fact that I assumed that firm entry in production stays at its long-run steady state value, i.e., $l_{\mathrm{P},t}=l_{\mathrm{P}}$.

The reader will note that the expression is almost identical to the closed economy one, except I have to sum over all countries and take into account that wages w_c, waiting times τ_c, adoption gaps z_c, and weights b_c are country specific.

A country-specific free entry condition holds

$$
\tilde{v}_{\mathrm{I},k,t}=\frac{1}{\gamma_k}h^{1-\lambda}_{\mathrm{F},k,t}s_{k,t},
$$

where a binding inequality is guaranteed as long as $\lambda<1$.

The evolution of the technological frontier follows from the free entry equilibrium and the entry technology

$$\begin{aligned}\frac{\dot{A}_{\mathrm{F}}^{\mathrm{W}}}{A_{\mathrm{F}}^{\mathrm{W}}} &= \sum_c \frac{A_{\mathrm{F},c}}{A_{\mathrm{F}}^{\mathrm{W}}} \frac{\dot{A}_{\mathrm{F},c}}{A_{\mathrm{F},c}} \\ &= \sum_c \chi_c \left\{ \frac{\gamma_c \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi} L h_{\mathrm{F},c}^{\lambda} b_c}{A_{\mathrm{F},c}} - \delta_{\mathrm{I}} \right\} \\ g_{\mathrm{F}}^{\mathrm{W}} &= \sum_c \frac{\gamma_c h_{\mathrm{F},c}^{\lambda} b_c}{a_{\mathrm{F}}^{\mathrm{W}}} - \delta_{\mathrm{I}}.\end{aligned}$$

The normalized technological frontier $a_{\mathrm{F}}^{\mathrm{W}} := \left(\frac{A_{\mathrm{F}}^{\mathrm{W}}}{L^{\frac{1}{1-\phi}}} \right)^{1-\phi}$ thus evolves according to

$$\begin{aligned} g_{a_{\mathrm{F}}^{\mathrm{W}}} &= (1-\phi)\, g_{\mathrm{F}}^{\mathrm{W}} - g_L \\ &= (1-\phi) \left\{ \frac{\sum_c \gamma_c h_{\mathrm{F},c}^{\lambda} b_c}{a_{\mathrm{F}}^{\mathrm{W}}} - \left(\delta_{\mathrm{I}} + \frac{g_L}{1-\phi} \right) \right\}. \end{aligned}$$

Note that this law of motion in the open economy is independent of the split of ownership of patents assumed at time zero, i.e., the share ζ does not show up.

Initial Conditions. In contrast, the evolution of country-specific normalized technology depends on ζ as follows. Note that at time $t \to 0$ where 0 demarcates the time when markets become integrated, the normalized frontier technology level in the West reads

$$\begin{aligned} \left(\frac{A_{\mathrm{F},0+\Delta}}{L^{\frac{1}{1-\phi}}} \right)^{1-\phi} &= \left(\frac{\zeta A_{\mathrm{F},0-\Delta}}{L^{\frac{1}{1-\phi}}} \right)^{1-\phi} \\ &= (\zeta)^{1-\phi} a_{\mathrm{F},0-\Delta}, \end{aligned}$$

and similarly the normalized level of frontier technology in the foreign country reads $(1-\zeta)^{1-\phi} a_{\mathrm{F},0-\Delta}$where $\Delta \to 0$. Note that both countries lose some of their ideas due to duplication.

Recall that I define the autarky frontier level relative to an initial real wage gap $\omega := \frac{w_{0-\Delta}}{w_{0-\Delta}^{*}}$, I use the following relationship to pin down the implied level of frontier technology in the East

$$A_{\mathrm{F},0-\Delta}^{*} = \frac{z_{0-\Delta}^{*}}{z_{0-\Delta}} A_{\mathrm{F},0-\Delta}$$

where $z_{0-\Delta}^{*}$ follows from solving the autarky equilibrium in the emerging market. Because I assume the emerging market has such a small research productivity in the integrated equilibrium, a counter-factually low level of income is implied. To fix this, I simply assume that copying works exactly like innovation except the fundamental productivity is higher than in innovation $\gamma^{*,copy} > \gamma^{*}$. It is easy to see that the allocation and the skill premium are unrelated to γ^{*}, which means I can simply solve the model for the low research productivity, and scale the frontier technology and real wage by the appropriate factor to obtain a wage gap ω. Some high level of $\gamma^{*,copy}$ will be exactly consistent with this wage gap.

Next, note that while the adoption gap in the advanced economy does not jump, the adoption gap in the emerging market changes discretely since it is defined relative to the frontier, which shifts out from the point of view of the emerging market. Formally,

$$z^*_{0+\Delta} = \frac{z^*_{0-\Delta} A^*_{\mathrm{F},0-\Delta}}{A^{\mathrm{W}}_{\mathrm{F},0+\Delta}}.$$

The evolution of this normalized technology level for either country over time changes slightly due to the global research externality

$$\frac{\dot{A}_{\mathrm{F},c}}{A_{\mathrm{F},c}} = \frac{\gamma_c h^{\lambda}_{\mathrm{F},c} b_c}{\chi_c a^{\mathrm{W}}_{\mathrm{F}}} - \delta_{\mathrm{I}},$$

where $\frac{A_{\mathrm{F},c}}{A^{\mathrm{W}}_{\mathrm{F}}} = \chi_c = \left(\frac{a_{\mathrm{F},c}}{a^{\mathrm{W}}_{\mathrm{F}}}\right)^{\frac{1}{1-\phi}}$ so $\chi_c a^{\mathrm{W}}_{\mathrm{F}} = (a_{\mathrm{F},c})^{\frac{1}{1-\phi}} \left(a^{\mathrm{W}}_{\mathrm{F}}\right)^{-\frac{\phi}{1-\phi}}$. In terms of normalized frontier growth rates I have

$$g_{a_{\mathrm{F},c}} = (1-\phi)\left[\frac{\gamma_c}{\chi_c}\frac{h^{\lambda}_{\mathrm{F},c} b_c}{a^{\mathrm{W}}_{\mathrm{F}}} - \left(\delta_{\mathrm{I}} + \frac{g_L}{1-\phi}\right)\right].$$

Capital Accumulation. Finally, note that the problem is simplified by assuming that a share of domestic final goods production is re-invested at a constant rate χ_s, which is the same across countries and consistent with the long-run equilibrium interest rate. This means that $1-\chi_s$ is the share of final output consumed.

Note that this share is sensitive to the initial allocation of patent ownership in the integrated equilibrium, ζ. At time zero, a share $Y^* \frac{\sigma-1}{\sigma} \alpha (1-\alpha) \zeta (1-\chi_s)$ of output in the East is consumed by the West, and vice versa a share $Y \frac{\sigma-1}{\sigma} \alpha (1-\alpha) \zeta (1-\chi_s)$ of output in the West is consumed by the East. This also means that some domestically used capital is now held by other countries. However, for the equilibrium dynamics of innovation and technology adoption this split is irrelevant. After understanding this simplification, the following algorithm applies.

Algorithm.

1. Guess $\{s_{c,t}\}$ for each country, holding $\{a^{\mathrm{W}}_{\mathrm{F},t}\}$ fixed.
2. Solve optimal adoption choice backward, and $\{\{\tilde{z}_{c,t}\}, \{r_{c,t}\}\}$ forward. Iterate until convergence for both countries.

(a) Compute $\{\tau, \tau^*\}$ and $\{w, w^*\}$ taking $\{z, z^*, k, k^*\}$ as given for each country.

(b) Solve innovator's problem

  i. Compute the value of innovation in each country recursively $\{v_{\mathrm{I}}, v_{\mathrm{I}}^*\}$

  ii. Implies equilibrium research effort based on free entry into research in each country $\{h_{\mathrm{F}}, h_{\mathrm{F}}^*\}$

  iii. Compute evolution of aggregate technological frontier using $\{h_{\mathrm{F}}, h_{\mathrm{F}}^*\}$.

  iv. Go back to i) and iterate up until $\left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{n+1} \approx \left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{n}$ where n stands for n^{th} iteration. Update the new guess gently $\left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{next} = relax * \left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{old} + (1 - relax) * \left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{new}$.

(c) Go back to a) and update $\{\tau, \tau^*\}$ and $\{w, w^*\}$ gently. Note that while I treat $\{z, z^*, k, k^*\}$ as fixed for now, waiting times and wages change since they also depend on the evolution of the technological frontier. Iterate till convergence. This always involves the inner loop i) – iv) and the outer loop a) – c).

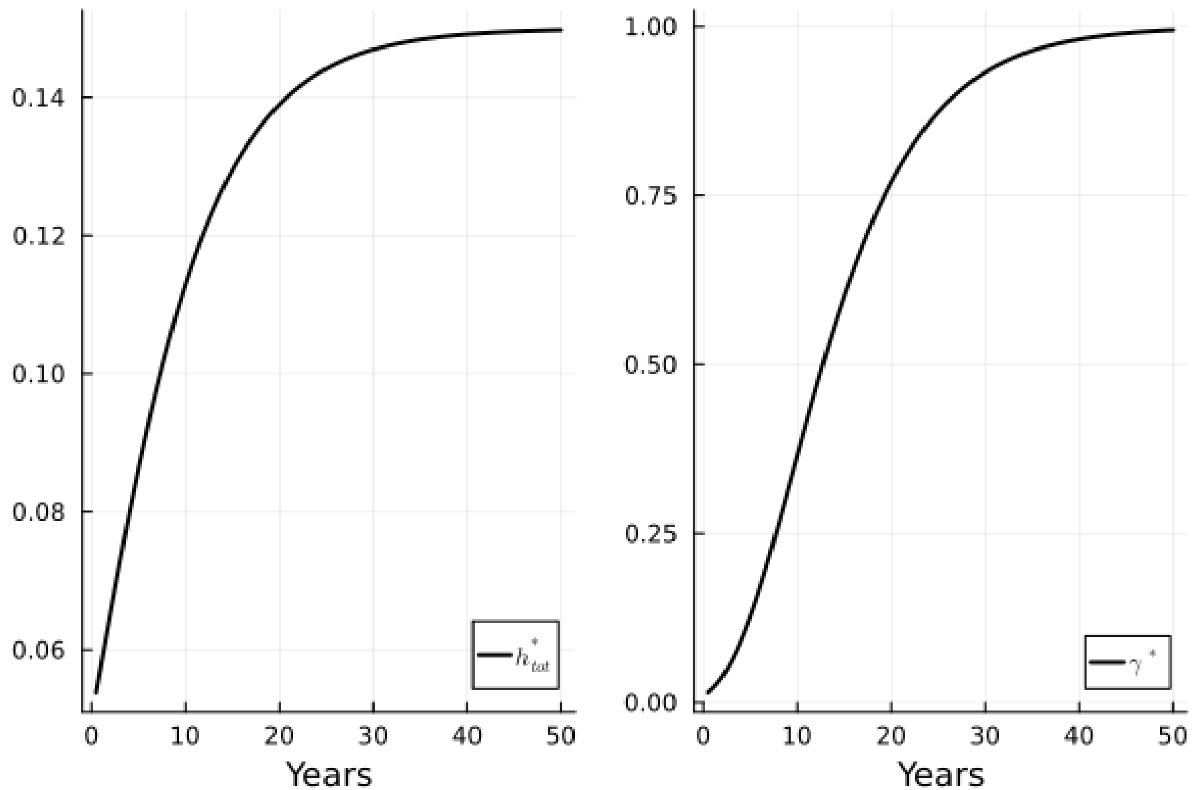
3. Given a new solution for the evolution of the technological frontier, return to 2. and iterate on all previous loops up until convergence in $\{z, z^*, k, k^*, a_{\mathrm{F}}^{\mathrm{W}}\}$.

4. Finally check skilled labor market clearing in each country, and use the excess demand function to update the skill premium in the right direction. Iterate over all loops up until the sequence of skill premia has converged in each country.

Full convergence. In the case of full convergence I let the research productivity and skill endowment of the poor country converge to the level of the rich country, i.e., ,$\gamma^* \rightarrow \gamma, h_{\text{tot}}^* \rightarrow h_{\text{tot}}$. I assume that the emerging market fully converges to the fundamentals of the advanced economy within 30 years. I impose a process that mimics convergence in the Solow model, and uses the formula $h_{\text{tot},t}^* = \left\{h_{\text{tot}}^{1-\alpha_{\text{conv}}}\left(1 - e^{-(1-\alpha_{\text{conv}})t}\right) + \left(h_{\text{tot},0}^*\right)^{1-\alpha_{\text{conv}}} e^{-(1-\alpha_{\text{conv}})t}\right\}^{\frac{1}{1-\alpha_{\text{conv}}}}$ where α_{conv} governs the speed of convergence, and an analogous expression applies to γ_t^*. I assume $\alpha_{\text{conv}} = .87$, which induces convergence patters as depicted in figure 16.

The previous computational algorithm applies almost unchanged. Changes in skilled labor endowments shows up in the market clearing condition. And the free entry condition into research needs to be updated slightly $\tilde{v}_{\mathrm{I},k,t} = \frac{1}{\gamma_{k,t}} h_{\mathrm{F},k,t}^{1-\lambda} s_{k,t}$ where the exogenous research productivity is now time varying for the emerging market. Laws of motion of idea creation also need to be updated to take account of the changing research productivity.

Figure 16. Full Convergence in East



C Extensions

C.1 Immigration

C.1.1 Emerging Market Contributing to the World Technological Frontier

The scenario considered here is arguably too bleak, and the most benevolent development would be one where the emerging market eventually contributes to the technological frontier. To formalize this scenario, suppose that $\gamma = \gamma^*$ and $h = h^*$ but $z > z^*$ i.e. the emerging market starts out of steady state but is otherwise identical to the advanced economy. I know the steady state solution provides productivity gains to both economies according to the constant elasticity $d \log w = \frac{1}{1-\phi} d \log L$, so a doubling of market size raises wages relative to trend by $2^{\frac{1}{1-\phi}} - 1 \approx 40\%$ for $\phi = -1$.

Initially, research takes a backseat in economy that is out of steady state, since returns to adoption are higher. In the long run symmetric equilibrium with same amount of research. Transition dynamics to be completed soon.

C.1.2 Different Sectoral Factor Intensity and Endogenous Labor Supply

In the baseline model I assume that production only requires capital and production labor, while adoption and innovation only requires skilled labor. This should be viewed as a simplified limiting case of a model where innovation requires a composite labor input $G_I(H, L)$ that is produced according to a constant returns to scale production function. Differentiating the cost function that pertains to G_I with respect to H leads to the amount of skilled labor needed to produce one unit of the composite good, denoted by b_I, see Feenstra (2015)'s introduction to the Heckscher-Ohlin theory of international trade. Assuming that $b_I > b_D > b_P$ is a useful generalization of the benchmark model so that each activity, innovation, adoption, and production, requires a mix of different labor types. I impose a strict ranking in terms of their factor intensity. Note that Heckscher-Ohlin theory and in particular the Rybczynski theorem would suggest an even stronger contraction in the production sector, but the gains from trade will be more broadly shared across worker types. Intuitively, this setting allows low skilled workers to benefit from gains in specialization in innovation.

Similar to the adjustment patterns in the model with composite labor goods, one can allow for an endogenous labor supply that will increase reallocation into innovation and ease the pressure on the skill premium. It would be easy, however, to extend the model by allowing workers to choose their education. One can incorporate this effortlessly into the market clearing condition for high-skilled labor (**??**) simply by letting the relative supply of skilled labor h^{tot} be a function of the skill premium $h^{tot} = h(s)$ s.t. $h'(s) > 0, h''(s) \geq 0$, and $h(1) = 0$.[95] Again, such a model offers more scope for production labor to gain from market integration.

[95] Micro-foundations to obtain an upward-sloping relative supply of skilled labor are plentiful, see for instance Acemoglu et al. (2018).

D Calibration

Calibrating λ. To calibrate the model I have to pin down λ,which does not have a direct antecedent in the literature. As explained in the main body of the paper the parameter calibrated in Jones (1995) is slightly different. In principal, cross-country research specialization among a set of countries with similar research productivities γ allows me to identify λ since the theory implies that the steady state share of ideas is a function of specialization in research, and total country size measured in terms of production labor

$$\log \chi_c = \alpha_0 + \lambda \log H_{\mathrm{F}} + (1-\lambda) \log L.$$

I use data from the OECD, and combine it with Barro and Lee's skill measure. I proxy the steady state patent share (a stock) with average patents over the period from 2011-2019 (a flow). Note that patents are an imperfect proxy that likely understate innovative activity and technological change. As long as the paten share is a linear function of the larger share of actual ideas develop in some country, the approach works nonetheless.

The measure of patents is based on patents captured by the Patent Cooperation Treaty (PIC), which are global patents that simultaneously protect intellectual property among several countries. This selection helps make patents comparable across countries, and is closest to the notion of "global ideas" in the paper. Clearly, patents are a crude measure of innovative activity, and there is no reason to believe the economy is exactly in steady state. Nor should we think that there is no heterogeneity in innovative productivity among rich countries in Europe. Overall, the approach is imperfect.

Perhaps surprisingly, the empirical results are extremely sensible without any massaging by using just a simple simple linear regression. After making an ad-hoc adjustment for the bias induced by unobserved heterogeneity in patenting productivity,[96] I arrive at an estimate $\lambda = .9$ which is a reasonable value for a first pass.

I provide robustness results next where I use total employment instead of production labor but the estimation is otherwise unchanged.

[96]Sampson (2023) argues heterogeneity in innovative productivity is a lot smaller among advanced economies relative to the gap between advanced economies and emerging markets.

Figure 17. Regression for λ

	log patents	log patents	log patents	log patents	log patents
log researchers	1.093***		0.980**		0.878**
	(0.0644)		(0.335)		(0.390)
log employment		1.070***	0.113	1.093***	0.122
		(0.0745)	(0.353)	(0.0657)	(0.390)
log (research/employment)				0.980**	
				(0.335)	
N	13	13	13	13	13
R^2	0.961	0.944	0.961	0.961	

The outcome variable is log patents. Employment is defined as total production labor L consistent with the theory. Robust standard error are computed, * p<0.10, ** p<0.05, *** p<0.01. The last column runs a constrained regression enforcing that the slope coefficients have to add up to one.

Table 2. Regression for λ robustness

	log patents	log patents	log patents	log patents
log researchers	1.093***		0.942**	
	(0.0644)		(0.360)	
log employment		1.074***	0.152	1.093***
		(0.0746)	(0.383)	(0.0654)
log (research/employment)				0.942**
				(0.360)
N	13	13	13	13
R^2	0.961	0.945	0.961	0.961

The outcome variable is log patents. The main difference to the previous table is that employment is now total employment. Robust standard error are computed, * p<0.10, ** p<0.05, *** p<0.01. The last column runs a constrained regression enforcing that the slope coefficients have to add up to one.