# Uneven Growth and Excess Savings along the Rural-Urban Transition Path*

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**Abstract**

Fast-growing emerging economies save at high rates and export capital, precisely when standard consumption smoothing arguments predict that they should borrow. I argue that uneven income growth during episodes of fast-paced structural transformation can resolve this puzzle. Households moving from traditional agriculture into the urban sector experience rapid income growth, but face uncertainty about their long-run urban earnings, and some end up worse off. Aggregate growth therefore substantially overstates the growth households expect when making consumption and saving decisions. Embedding this logic into a model of long-run rural-urban structural change generates realistic transition dynamics and strengthens the mechanism quantitatively: reallocation across sectors generates aggregate growth whenever rural-urban wage gaps are large, without creating corresponding consumption-smoothing pressure. I develop this argument in a tractable model that delivers the transition of the income distribution in closed form. Calibrated to China, the model reproduces the observed path of income inequality and generates excess saving despite exceptionally fast aggregate growth, with capital outflows peaking at around seven percent of GDP along the transition path.

*Keywords:* Excess Saving; Capital Flows; Structural Transformation; Precautionary Savings; Income Risk; Inequality; Rural–Urban Transition

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# 1 Introduction

In a seminal article, Lucas (1990) asks: "Why doesn't capital flow from rich to poor countries?" The puzzle is particularly stark during growth miracles. Fast-growing emerging economies should borrow against their much higher future income, yet they instead save in excess of domestic investment and export capital. As Gourinchas and Jeanne (2013) show, this is primarily a saving puzzle: capital flows in the wrong direction not because investment fails to respond to growth, but because saving rises in excess of investment. This behavior is difficult to reconcile with a cornerstone of modern macroeconomics, the permanent income hypothesis. Households that are relatively poor today but expect their income to grow rapidly should smooth consumption by borrowing against future income and run current account deficits during the catch-up phase. Yet the opposite pattern characterizes major growth miracles, including Japan, Taiwan, Korea and, most recently, China.

The difficulty in explaining this pattern is the strong consumption-smoothing force when countries catch up with the rest of the world. In the benchmark neoclassical model, rapid growth induces households to borrow against higher future income to smooth consumption. Introducing idiosyncratic risk generates precautionary savings, but does not eliminate this force. In canonical incomplete-markets models, households face idiosyncratic shocks around a common aggregate growth trend and therefore still expect to become substantially richer as the economy converges. Precautionary savings must consequently be very strong to dominate the consumption-smoothing motive during a growth miracle (Coeurdacier, Rey, and Winant, 2019).

The main idea of this paper is to introduce uneven and risky income growth in the urban sector of a dual-sector model, in contrast to benchmark models in which every household grows at the same rate. Households moving from traditional agriculture into the urban sector initially experience rapid income growth, but face uncertainty about their long-run urban earnings. Some remain on a high-growth path for many years and move far up the income distribution, while others do not and can end up worse off. Aggregate growth is therefore disproportionately driven by households in the emerging upper tail and substantially overstates the growth that an individual household expects when making consumption and saving decisions. This weakens the consumption-smoothing motive by enough that the risk of ending up worse off can dominate it, generating precautionary saving at the household level and excess savings and capital outflows in the aggregate. Quantitatively, uneven growth reduces the income variance required to overturn the smoothing motive by a factor of sixteen relative to a model in which catch-up growth is distributed evenly across households.

Embedding this mechanism of uneven and risky growth into a model of rural–urban structural change is essential in two further dimensions. First, reallocation across sectors generates aggregate growth whenever rural–urban wage gaps are large, without creating a corresponding consumption-smoothing pressure. The migration decision satisfies an equilibrium indifference condition: the value of moving to the urban sector equals the value of remaining rural despite the higher urban wage. Growth generated by reallocation across sectors therefore does not represent an unexploited increase in expected lifetime income against which households want to borrow. Second, the continuous flow of households from the rural into the urban sector generates realistic transition dynamics. New migrants enter the high-growth and risky phase of their urban careers while earlier cohorts gradually leave it, creating a prolonged and eventually declining precautionary savings pressure rather than a one-time response.

I proceed in three steps. First, I document the empirical patterns that motivate the theory. Across emerging markets, the negative relationship between productivity growth and capital inflows documented by Gourinchas and Jeanne (2013) is concentrated among countries undergoing fast-paced rural–urban structural change. I then focus on China, a fast-growing emerging economy with high saving rates and persistent capital outflows, and use household panel data to document that urban households hold substantially more assets relative to income than rural households. Moreover, urban households not only hold larger buffers but continue to accumulate them faster than rural households over the panel. These patterns are consistent with the accumulation of buffer-stock savings as households undergo a risky sectoral transition.

In the second step, I develop a tractable model of miracle growth that emphasizes the risky and uneven nature of the rural–urban transition. The environment is standard in many respects: infinitely lived households with conventional preferences have perfect foresight over aggregate dynamics. I depart from the benchmark neoclassical model in two crucial ways. First, the economy consists of a traditional rural and a modern urban sector, with continuous structural change driven by productivity growth in the urban sector.[1] Second, income growth in the urban sector is risky and uneven. New urban households initially experience fast income growth, but the duration of this phase is uncertain and households do not know ex ante where they will eventually end up in the urban income distribution. I interpret this uncertainty as reflecting differences in the returns to urban-specific human capital that are only revealed over the course of an urban career. The formulation keeps two sources of dispersion distinct. The random duration of the high-growth phase makes catch-up growth uneven across households; a permanent income draw adds dispersion in long-run income levels and allows some households to end up worse off.

A key contribution of the paper is that this environment remains analytically tractable. I solve the rural–urban migration decision and the evolution of the income distribution analytically. The analytical solution makes it possible to attribute the results to specific primitives rather than infer the mechanism from a numerical experiment. In particular, the model delivers a simple condition that determines when precautionary savings dominate consumption smoothing and the economy exports capital along the transition path. It also makes clear why uneven growth alone is not sufficient: uncertainty about how much better off a household will become does not generate the required precautionary motive; households must face the possibility of ending up worse off.

In the third step, I calibrate the model to China and explore the transition dynamics. The two parameters governing idiosyncratic risk are disciplined by the size of the growth episode and long-run income dispersion; neither is chosen to match a saving rate, current account, or asset ratio. The model reproduces the observed rise in Chinese income inequality between 1988 and 2013, generates a hump-shaped path for aggregate saving, and delivers current account surpluses peaking at around seven percent of GDP along the transition. A comparison with an even-growth version of the model illustrates the quantitative importance of uneven growth. Holding aggregate convergence fixed but letting all households grow at a common rate, the log variance of the permanent income draw required to generate excess savings rises from $0.30$ to $4.75$. The calibrated value of $0.49$, disciplined by long-run income dispersion rather than chosen to generate capital outflows, comfortably exceeds the lower threshold. Both economies contain the same source of permanent downside risk; what differs is whether catch-up growth itself is distributed evenly or unevenly across households.

The mechanism brings together elements that have appeared separately in several literatures. Lucas

[1] Throughout I use "rural", "agricultural" and "countryside" interchangeably, and likewise "urban", "non-agricultural" and "city". The model has two sectors, while the data measure sometimes one distinction and sometimes the other: the structural change series is agricultural employment, and the household data classify households as rural or urban. The two do not coincide exactly in China, where hukou status, location and occupation can differ, but nothing in the argument turns on the distinction.

(2004) models rural–urban migration and human capital accumulation, but without income risk or implications for capital flows. Harris and Todaro (1970) is an important precursor in emphasizing urban risk and the rural–urban wage gap, but the model is static and has no saving decision. Buera and Shin (2017) study structural transformation and capital flows without the rural–urban migration margin emphasized here, while Song, Storesletten, and Zilibotti (2011) generate capital outflows through reallocation across firm types and financial frictions rather than household income risk across sectors. My novel contribution is not a new precautionary-saving motive, but to show analytically how uneven growth can make a standard one powerful enough to overturn the consumption-smoothing force during a growth miracle. Embedding this mechanism in a model of rural–urban structural change then generates realistic transition dynamics for aggregate savings and capital flows.

**Literature Review**. The paper builds on the literature on precautionary savings and incomplete markets (Bewley, 1977; Deaton, 1991; Imrohoroğlu, 1989; Huggett, 1993; Aiyagari, 1994; Carroll, 1997). The paper is related to Carroll and Jeanne (2009), who study the role of unemployment risk in generating savings in fast-growing economies. Similarly, Coeurdacier, Rey, and Winant (2019) study the interaction between aggregate risk, precautionary saving, and capital flows in a two-country neoclassical growth model. In the Chinese context, Chamon and Prasad (2010) and He et al. (2018) highlight the link between rising income risk and precautionary savings. In contrast, I explore how growth-rate heterogeneity across households weakens the consumption-smoothing motive during a growth miracle. Heterogeneous income growth also connects the paper to Gabaix et al. (2016), who show that growth-rate heterogeneity can generate rapid increases in inequality at the upper tail of the income distribution. This feature is particularly relevant for China, where tail inequality increased rapidly over a period of less than two decades (Piketty, Yang, and Zucman, 2019).

The dual-sector structure connects the paper to the literature on rural–urban migration and structural transformation. Harris and Todaro (1970) emphasizes the interaction between rural–urban wage gaps and urban risk, while Lucas (2004) studies a rural–urban transition driven by human-capital accumulation. The mechanism here combines these elements in a dynamic saving problem: households endogenously migrate and subsequently face uneven and risky income growth in the urban sector. Rural–urban structural change matters because reallocation across sectors raises aggregate income whenever wage gaps are large, without generating corresponding consumption-smoothing pressure, while the continuous flow of migrants moves new households into the risky phase of their urban careers and generates realistic transition dynamics. The wage gap itself connects to the literature on agricultural productivity gaps (Caselli, 2005; Restuccia, Yang, and Zhu, 2008; Gollin, Lagakos, and Waugh, 2013). The assumption that risk differs across sectors is also consistent with evidence of substantial consumption insurance in rural economies (Townsend, 1994; Rosenzweig and Stark, 1989) and large uninsurable income risk in modern and fast-growing economies (Chamon, Liu, and Prasad, 2013; Santaeulalia-Llopis and Zheng, 2018).

Finally, the paper contributes to the literature on global imbalances. Gourinchas and Jeanne (2013) show that the tendency of faster-growing developing economies to run current account surpluses is driven primarily by excess savings relative to investment. Existing explanations emphasize, among other mechanisms, shortages of safe assets and financial development (Caballero, Farhi, and Gourinchas, 2008; Mendoza, Quadrini, and Rios-Rull, 2009), which largely abstract from transitional catch-up growth dynamics. Domestic financial frictions and firm-level misallocation have also been emphasized by Song, Storesletten, and Zilibotti (2011) and Buera and Shin (2017). Relative to these works, I focus on household saving and its interaction with the risky rural–urban transition. An important result is that uneven growth alone is not sufficient to generate excess savings: it weakens the consumption-smoothing motive, but precautionary savings require that households also face the possibility of ending up worse off. This result is established particularly cleanly here, but the underlying insight is more general: whenever economic transitions create winners and losers, households exposed to the downside of reallocation have an incentive to accumulate precautionary savings.

The rest of the paper proceeds as follows. Section 2 documents the empirical patterns motivating the mechanism. Section 3 develops the model and characterizes the interaction between uneven growth, structural change, and precautionary savings. Section 4 disciplines the model quantitatively and studies inequality, excess savings, and capital flows along the transition path. Section 5 concludes.

## 2 Miracle Growth and Savings in China

In this section I provide a set of stylized facts relating to the macro as well as the micro dynamics of miracle growth. While the macro facts of growth, savings, and capital flows are partially known, I relate them to fast-paced structural change and show that the "allocation puzzle" (Gourinchas and Jeanne, 2013) is concentrated among emerging markets exhibiting an exceptionally fast reallocation of workers from the rural to the urban sector. I then focus on China and analyze household level panel data to provide micro evidence consistent with urban-rural differences in saving behavior.

### 2.1 Growth, Capital Flows, and Structural Change

Figure 1 is a version of the main figure in the influential paper of Gourinchas and Jeanne (2013), which revisits the negative link between productivity growth and capital inflows. While the left panel plots the familiar puzzling negative relationship between productivity growth and capital inflows, the right panel separates countries into economies that exhibit relatively fast or relatively slow structural change. In particular, the orange diamonds represent economies that display above average declines of the agricultural employment share. Comparing the left and right panels, the negative relationship is present only among countries experiencing fast structural change out of agriculture, where the correlation is $-0.82$, and is absent among the rest, where it is $0.02$. This pattern suggests that structural change is not merely correlated with capital outflows, but may fundamentally alter household saving incentives during catch-up growth.

The pattern is particularly pronounced among the so-called "miracle economies" (Lucas, 1993), including the East Asian tigers—Japan, Korea, and Taiwan—and, more recently, China. I focus on China for the remainder of the paper. It is the largest episode of this kind, a fast-growing emerging economy with high saving rates and persistent capital outflows, and the one for which household panel data allow the mechanism to be examined directly.

Figure 2 shows the Chinese transition. The agricultural employment share falls from roughly 70 percent in the early 1980s to below 30 percent by 2017, while GDP per capita rises from about five to 27 percent of the US level. Over the same period the gross saving rate rises from 33 percent of GDP to a peak of 51 percent around 2010 and subsequently declines, and China becomes a persistent net capital exporter. The saving rate displays a pronounced hump-shaped pattern, and it emerges during a period of sustained rural–urban reallocation rather than after it: the agricultural share is still falling rapidly when saving turns down. The model developed below accounts for the timing through the changing composition of the urban population rather than through the pace of reallocation alone. The figure also shows how far apart the two saving

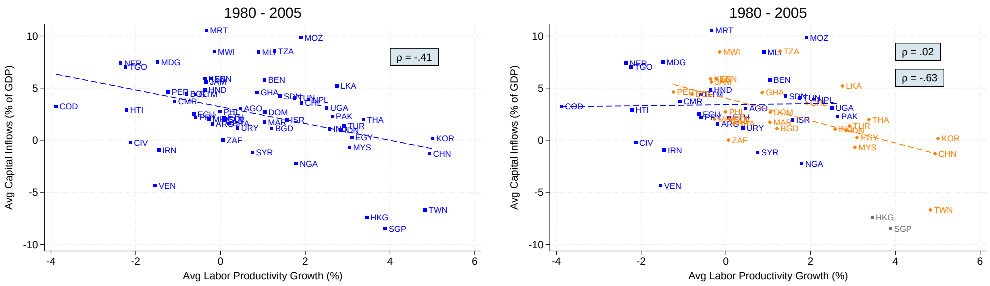


Figure 1. Relationship between average capital inflows and average labor productivity growth for a cross section of emerging markets following Gourinchas and Jeanne (2013). Orange diamonds represent emerging markets with above average decline of the agricultural employment share. Current-account-to-GDP is taken from WDI. Agricultural employment shares are taken from the GGDC ten sector database, and WDI. Labor productivity growth rates are computed from real national series from the PWT 9.1. Hong Kong and Singapore are dropped when computing the regression lines for the split sample due to their special role as financial centers. The correlation between capital inflows and productivity growth for the full sample is $-.41$. For the split sample, the correlation is $-.63$ and $0.02$ for countries with fast and slow structural change, respectively.

measures are. Gross saving reaches roughly half of GDP, while the current account, which measures saving in excess of domestic investment, peaks at seven percent in 2007 and falls back thereafter as investment catches up.

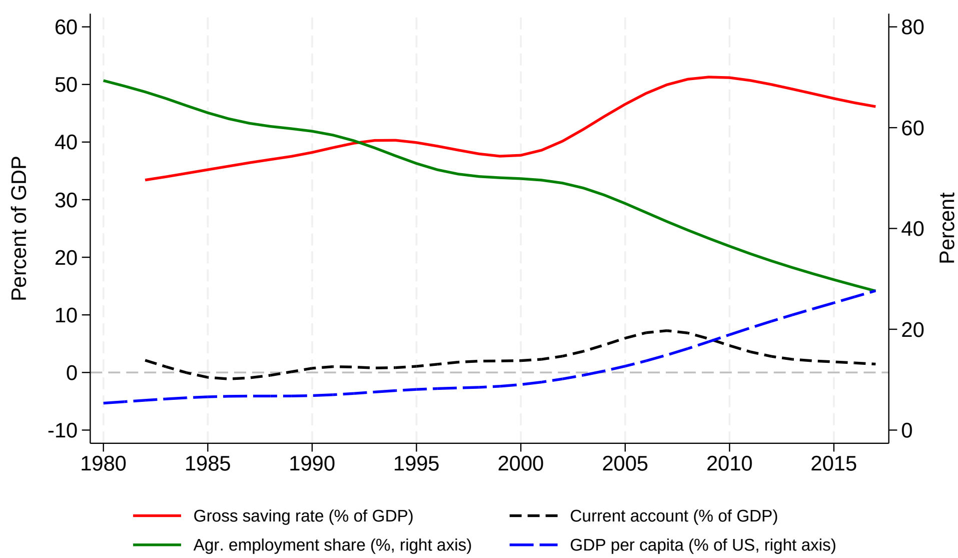


Figure 2. The Chinese transition. Gross saving and the current account are measured as a share of GDP on the left axis; the agricultural employment share and GDP per capita relative to the United States are measured in percent on the right axis. Saving and current account data are from the WDI. The agricultural employment share is from the GGDC ten sector database, extended after 2011 using changes in the WDI series. GDP per capita relative to the United States is measured at PPP from the PWT 9.0, extended after 2014 using national accounts growth rates from the PWT 9.1. All series are smoothed using an hp-filter with smoothing parameter of 8.5.

In the benchmark neoclassical model, rapid catch-up growth implies consumption smoothing, low saving rates, and large current account deficits.[2] These predictions are not borne out in the data. It is worth

[2]Convergence dynamics in the benchmark model tend to be monotone, see the textbook treatment by Barro and Sala-i-Martin

being precise, however, about which object the model speaks to. Chinese gross saving reaches roughly half of GDP, and nothing in what follows accounts for that level. The object of interest is saving in excess of domestic investment, which corresponds to the current account and is where Gourinchas and Jeanne (2013) locate the allocation puzzle. Their decomposition attributes the puzzle primarily to the saving rather than the investment margin. The model below therefore focuses on the forces generating excess saving during rapid catch-up growth rather than on the level of gross saving itself.[3]

## 2.2 Urban-Rural Differences in Asset Accumulation

I next document differences in asset accumulation across rural and urban households in China using the China Family Panel Studies (CFPS), a nationally representative household panel with detailed information on income, expenditures, assets, and demographics. The goal is to establish whether the rural–urban margin emphasized by the theory is reflected in actual household saving behavior.

I focus on asset-to-income ratios following Carroll and Samwick (1997). Assets are stock variables that summarize accumulated saving and are less noisy than flow measures of saving (Dynan, Skinner, and Zeldes, 2004). Moreover, in buffer-stock models, precautionary wealth is proportional to permanent income, making the asset-to-income ratio a natural statistic for studying precautionary saving (Carroll, 1997). The same relationship holds in the model developed in the next section. I therefore average household assets and income over the sample period so that income is closer to a measure of permanent income.[4] Income is measured as net labor income, including business, agricultural, and wage income after taxes and transfers. I consider both total assets—net household wealth including housing, business assets, and durables—and safe assets, defined as financial assets such as deposits, bonds, stocks, and cash. I focus on safe assets because they are particularly relevant for the international capital outflows of emerging markets (Bernanke et al., 2005; Aguiar and Amador, 2011; Gourinchas and Jeanne, 2013). The sample is restricted to households whose head is of working age, observed in at least three waves, and who remain in the same sector throughout, so that differences in asset accumulation are not driven by life-cycle effects, household splits, or sectoral moves within the sample.

Figure 3 shows the median asset-to-income ratios for rural and urban households. Urban households hold substantially more safe assets relative to income: the median ratio is about $0.6$, compared with $0.25$ for rural households, despite urban income being more than twice as high. The difference is therefore not a consequence of higher urban income; urban households accumulate substantially larger asset buffers relative to what they earn.

The rural–urban difference does not by itself establish a causal effect of urban residence. Nevertheless, the gap remains large after controlling for age, education, household size, and other demographics, as reported in Table 1. Results for total assets and estimates using sampling weights are reported in the appendix and yield similar differences.[5] This is consistent with a stronger precautionary saving motive among urban households.

(2003). If convergence is driven by TFP rather than capital accumulation, the aggregate saving rate tends to fall due to strong income effects for standard preferences. One classic explanation is consumption habit, see Carroll, Overland, and Weil (2000); however, consumption dynamics in the micro data seem to contradict this explanation, see Chamon and Prasad (2010).

[3]This also justifies abstracting from capital accumulation dynamics by maintaining a small open economy setup with a fixed interest rate; see Section 6.4 for an extension with capital. There is also an important distinction between safe assets and more risky FDI flows, as well as private versus public flows, see Aguiar and Amador (2011). My framework focuses on safe assets, and I abstract away from the distinction between private and public flows.

[4]Appendix 7.1 provides details on variable construction, sample selection, and summary statistics.

[5]Since income is in the denominator and urban households tend to have higher income, controlling for income increases the estimated rural–urban difference.

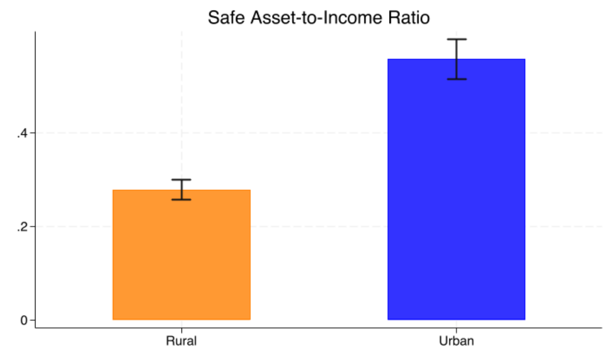


Figure 3. Cross sectional rural-urban median differences in safe asset-to-income ratios based on CFPS with 95% confidence intervals.

Table 1. Rural-Urban Median Differences for Safe Assets

| | (1)<br>safe_asset_income_ratio | (2)<br>safe_asset_income_ratio | (3)<br>safe_asset_income_ratio |
|---|---|---|---|
| urban dummy | 0.279*** | 0.264*** | 0.168*** |
| | (0.0214) | (0.0198) | (0.0229) |
| _cons | 0.278*** | -0.441** | -0.506** |
| | (0.0106) | (0.207) | (0.225) |
| age | No | Yes | Yes |
| sex and hh size | No | No | Yes |
| education | No | No | Yes |
| Observations | 5957 | 5957 | 5945 |

*Note*: The dependent variable is the household financial asset-to-income ratio. This contains bank deposits, stocks, derivatives, bonds, cash, and other financial assets. Robust standard errors in parentheses. *, **,*** denote statistical significance at 1, 5, and 10 percent level.

The cross-sectional gap documented above is a difference in stocks. The panel dimension of the CFPS allows me to ask whether it is also a difference in flows, which is what the model implies: urban households, exposed to uninsurable income risk, should not only hold larger buffers but keep adding to them, while rural households should not. For each household I compare the first and the last wave in which it appears and compute the change in assets per year, scaled by the household's average income over the panel. This measures the share of one year's income that a household adds to its assets annually; using average rather than single-wave income reduces measurement error in the denominator and avoids mechanical mean reversion a first-wave denominator would induce.

Table 2 reports the result. Urban households add about five percent of annual income to their financial assets each year relative to rural households. The median gap remains significant at 2.8 percentage points even after controlling for demographics, income, income growth and initial asset holdings. An appendix table repeats the exercise using the inverse hyperbolic sine of the asset-to-income ratio and obtains the same answer. Results for total net assets are larger still and are reported in the appendix.[6]

**Evidence on Income Risk.** The Chinese growth miracle is also associated with substantial household income risk (Chamon, Liu, and Prasad, 2013; He et al., 2018; Santaeulalia-Llopis and Zheng, 2018). Two features are particularly relevant for the mechanism developed below. First, income growth is heterogeneous across households rather than well described by a common long-run growth rate. This heterogeneity is reflected in the rapid increase in upper-tail inequality during China's growth transition (Piketty, Yang, and

[6]The mechanism runs through households entering the urban sector, which invites the question of whether recent entrants can be identified directly. The CFPS panel is not well suited to this. Only 443 of 6,463 households are observed changing sector between their first and last interview, and most of these are not migrants: their community is reclassified from rural to urban while the household stays in place.

Table 2. Rural-Urban Differences in the Rate of Asset Accumulation

| | (1) median | (2) median | (3) median | (4) OLS |
|---|---|---|---|---|
| urban dummy | 0.0516*** | 0.0388*** | 0.0281*** | 0.0663*** |
| | (0.00525) | (0.00589) | (0.00614) | (0.00993) |
| _cons | 0.0210*** | 0.0569 | -0.0408 | 0.173 |
| | (0.00182) | (0.0468) | (0.0547) | (0.125) |
| demographics | No | Yes | Yes | Yes |
| income | No | No | Yes | Yes |
| initial assets | No | No | Yes | Yes |
| Observations | 5579 | 5389 | 4872 | 4708 |

*Note*: The dependent variable is the change in household financial assets between the first and the last wave in which the household is observed, divided by the number of years elapsed and by the household's average income over the panel. It measures the share of one year's income added to financial assets each year. There is one observation per household. Columns 1 to 3 are median regressions; column 4 is OLS with the dependent variable trimmed symmetrically at the 2nd and 98th percentile. Demographics are age, age squared, gender, the log of the number of adults, and years of schooling. Income controls are log income in the first wave and annualised income growth. Initial assets are an indicator for holding no financial assets in the first wave together with the initial asset-to-income ratio. Households holding no financial assets are retained throughout. Robust standard errors in parentheses. *, **, *** denote statistical significance at the 10, 5, and 1 percent level.

Zucman, 2019). Gabaix et al. (2016) point out that such a quick evolution of top income inequality cannot be obtained with standard idiosyncratic income shocks but is consistent with growth rate heterogeneity. Second, the rural–urban transition is an important source of uncertainty about long-run earnings. Migrants move into a sector with much higher average wages but face substantial uncertainty about where they will end up in the urban income distribution, and some do not experience sustained income gains. Moreover, Baseler (2023) and Porcher, Morales, and Fujiwara (2024) provide evidence on information frictions associated with migration in developing economies. In companion work (Trouvain, 2026), I estimate a structural model of rural–urban migration in South Africa and find that permanent income risk is higher in the urban than in the rural sector, informal insurance is weaker in urban areas, and households have limited ex ante information about their urban earnings.

# 3 A Stylized Two-Sector Model of Uneven Miracle Growth

This section presents a tractable model that isolates the mechanism emphasized in the data: (i) structural change (rural $\to$ urban reallocation) together with (ii) risky, uneven household-level catch-up growth generate strong precautionary saving motives, a hump-shaped aggregate saving rate, and net capital outflows. The model is intentionally stylized so that the core mechanism is explicit.

## 3.1 Environment

Time is continuous, $t \geq 0$. A unit mass of infinitely lived households indexed by $i \in [0,1]$ supply labor inelastically. There are two sectors: rural and urban. Rural households decide on whether to permanently migrate to the urban sector. Rural households are hand-to-mouth as in Moll (2014) while urban households solve a forward-looking consumption-saving problem. Let $L_t^{\mathrm{R}}$ and $L_t^{\mathrm{U}}$ denote the share of households in rural and urban sector so $L_t^{\mathrm{R}} + L_t^{\mathrm{U}} = 1$. At time zero a share $L_0^{\mathrm{U}} = 1 - L_0^{\mathrm{R}}$ of households already lives in the city. Of these, $M_{1,0}$ have drawn their type and $M_{0,0} = L_0^{\mathrm{U}} - M_{1,0}$ are still in the high-growth phase. The households that have already drawn carry whatever income dispersion the pre-reform urban sector had, denoted $\sigma_0^2$. This is an initial condition rather than a parameter of the income process: those households hold no assets and never solve the saving problem below, so $\sigma_0^2$ enters the cross-sectional income distribution and nothing else.

**Technology and Market Structure.**

Aggregate output equals the output of each sector, $Y_t = Y_t^{\mathrm{R}} + Y_t^{\mathrm{U}}$, where the price is normalized to one and each sector produces the same final output good. Rural production exhibits diminishing returns due to fixed land (normalized to one)

$$Y_t^{\mathrm{R}} = A_0^{\mathrm{R}}(L_t^{\mathrm{R}})^{\chi}, \qquad \chi \in (0,1), \tag{1}$$

and rural per-worker compensation (including implicit land return) is $w_t^{\mathrm{R}} = A_0^{\mathrm{R}}(L_t^{\mathrm{R}})^{-(1-\chi)}$, in the spirit of Lewis (1954) where households receive their average, not marginal, product, and there is no heterogeneity across households as each of them is endowed with one efficiency unit of labor.

Urban production is constant-returns-to-scale in effective labor with productivity $A_t^{\mathrm{U}}$

$$Y_t^{\mathrm{U}} = A_t^{\mathrm{U}} \int_{i \in L_t^{\mathrm{U}}} h_t^i \, di, \tag{2}$$

where $h_{i,t}$ are the efficiency units of labor of household $i$ in urban activity. Production is perfectly competitive so that households earn their marginal product and the wage per efficiency unit equals $w_t^{\mathrm{U}} = A_t^{\mathrm{U}}$.

**Preferences, Budget Constraint, and Migration.**

Households have CRRA flow utility with risk aversion $\eta > 0$ and discount rate $\rho$. Rural households consume their labor income each instant: $c_t^{\mathrm{R}} = w_t^{\mathrm{R}}$. Rural households face a migration decision. They are free to leave the rural sector, and do so whenever the expected value in urban exceeds the value of staying in the rural sector net of a utility cost of migration $\tau$. The optimality conditions have to satisfy the following two equations

$$\rho V_t^{\mathrm{R}} = \frac{(w_t^{\mathrm{R}})^{1-\eta}}{1-\eta} + \dot{V}_t^{\mathrm{R}}, \qquad V_t^{\mathrm{R}} \geq \frac{V_t^{\mathrm{U}}}{\tau^{1-\eta}}. \tag{3}$$

where $\tau^{1-\eta}$ scales the utility cost appropriately such that $\tau > 1$ can be interpreted as a wedge in consumption equivalents. Note that rural households enter the urban sector with zero assets.

Urban households choose consumption $c_t$ and asset holdings $b_t$ to maximize utility

$$V_t^{\mathrm{U}} = \max_{\{c_s\}} \mathbb{E}\left[\int_t^{\infty} e^{-\rho(s-t)} \frac{c_s^{1-\eta}}{1-\eta} \, ds\right], \qquad \dot{b}_s = rb_t + y_t - c_t, \tag{4}$$

subject to a standard intertemporal budget constraint and the no-Ponzi condition where $b_t$ represents holdings of a risk-free internationally traded bond that yields return $r$, and $y_t$ and $c_t$ denote income and consumption. The economy is small relative to the rest of the world (ROW) so the interest rate is exogenous.

Aggregate asset accumulation, i.e., aggregate savings, in the small open economy equals

$$\int_{i \in L_t^{\mathrm{U}}} \dot{b}_{i,t} \, di = Y_t^{\mathrm{U}} + rB_t - C_t^{\mathrm{U}}, \tag{5}$$

where $B_t$ are aggregate bond holdings.

**Growth and Risk.**

The productivity of the urban sector after the emerging market "reforms" at time zero in ways left unspecified grows at rate $g$, which is the same as for the ROW

$$A_t^{\mathrm{U}} = A_0^{\mathrm{U}} e^{gt}. \tag{6}$$

Given a fixed factor in the rural sector, productivity growth in the urban sector induces structural change consistent with the importance of pull-factors during early stages of the development process (Alvarez-Cuadrado and Poschke, 2011; Hnatkovska and Lahiri, 2018).[7]

A household that migrates at time $t_{m,i}$ enters a temporary high-growth phase during which its (log) income grows at a rate $g_h > g$. The duration of this high-growth phase is stochastic, with households exiting the fast-growth process at a Poisson arrival rate $\lambda$. Upon exit at time $T_i$, the household draws a multiplicative type $\varphi_i > 0$ from distribution $F(\varphi)$, which I interpret as reflecting differences in the returns to urban-specific human capital that are only revealed over the course of an urban career. Following this draw, the household's income grows at the ROW rate $g$ and is subject to no further idiosyncratic uncertainty. Log income thus equals

$$\log y_t^i = \begin{cases} \log A_t^{\mathrm{U}} + (g_h - g)(t - t_{m,i}), & t_{m,i} \le t \le T_i, \\ \log A_t^{\mathrm{U}} + (g_h - g)(T_i - t_{m,i}) + \log \varphi_i, & t > T_i. \end{cases}$$

The same process can be written in efficiency units. A household enters the urban sector with $h^i_{t_{m,i}} = 1$ and accumulates urban-specific human capital at rate $g_h - g$ while in the high-growth phase. The type draw on exit multiplies the human capital it has accumulated by $\varphi_i$,

$$h_t^i = \begin{cases} e^{(g_h - g)(t - t_{m,i})}, & t_{m,i} \le t \le T_i, \\ \varphi_i \, e^{(g_h - g)(T_i - t_{m,i})}, & t > T_i, \end{cases}$$

so that $y_t^i = w_t^{\mathrm{U}} h_t^i = A_t^{\mathrm{U}} h_t^i$. Income growth in excess of $g$ comes from human capital accumulation rather than from wage growth, and $\varphi$ is a draw over the return to the human capital a household has built in the city.

**Remark.** The stochastic process implies exponential, memoryless waiting times in the high-growth regime, which keeps the distributional dynamics tractable. This structure closely resembles a "Luttmer rocket" (Luttmer, 2011), here applied to household income growth rather than firm size. The formulation keeps two sources of dispersion distinct: uneven growth during the high-growth phase, and permanent dispersion in income levels afterwards. It also captures a feature of structural transformation that motivates the exercise. Human capital differences are largely irrelevant in agriculture but first-order in urban production, so ex ante similar households can experience very different outcomes once they move.

**Competitive Equilibrium.**

A competitive equilibrium consists of prices $\{w_t^{\mathrm{R}}, w_t^{\mathrm{U}}, r\}$, rural and urban shares $\{L_t^{\mathrm{R}}, L_t^{\mathrm{U}}\}$ consumption allocations and income $\{c_{i,t}, y_{i,t}\}$, migration decisions $\{t_{i,m}\}$ and joint time-varying distribution over $(t_m, T, b, \varphi)$ such that households optimize given prices and stochastic processes, firms maximize profits, labor markets clear, and the joint distribution evolves consistently with individual decisions and the stochastic process. This equilibrium is non-stationary with ever declining rural share.

## 3.2 Solving the Model

The model can be solved backwards, focusing on the urban sector first, and then analyzing the rural migration problem. Note that the ROW grows at rate $g$ and all uncertainty on the household level is already

[7] See Herrendorf, Rogerson, and Valentinyi (2014) for a discussion of a variety of models that give rise to structural change, which depends on both the preferences structure of the agents (homothetic vs non-homothetic and complements vs substitutes) as well as the direction of technological change (productivity growth in the city vs productivity growth in the rural sector).

resolved, so that the standard Euler equation implies $r = \eta g + \rho$. It is convenient to define $\kappa \equiv r - g = (\eta - 1)g + \rho$, which appears throughout.

**Urban household problem.**

After the type draw ($t \geq T$) the household faces deterministic growth at rate $g$ and no idiosyncratic risk, so consumption growth obeys the standard Euler equation, $\frac{\dot{c}_t}{c_t} = \frac{1}{\eta}(r - \rho) = g$. Using this, the post-draw value function equals

$$V^{\mathrm{U}}(b_t, y_t(\varphi, T)) = \frac{\left(y_t + \kappa b_t\right)^{1-\eta}}{(1-\eta)\,\kappa}, \tag{7}$$

where $y_t(\varphi, T)$ makes clear that income depends on both the type draw $\varphi$ as well as the time spent in the high-growth regime, $T$. Whilst the household is in the high-growth regime, I show in Appendix 6 that the household value function equals

$$V_t^{\mathrm{U}} = \max_{\{c_s\}_{s \geq t}} \int_t^{\infty} e^{-(\rho+\lambda)(s-t)} \left( \frac{c_s^{1-\eta}}{1-\eta} + \lambda\, \mathbb{E}_{\varphi}\left[V^{\mathrm{U}}\left(b_s, \varphi y_s\right)\right] \right) ds, \tag{8}$$

where $V^{\mathrm{U}}(\cdot)$ inside the expectation is the post-draw value function in (7), evaluated at the assets the household carries into the draw and at its income immediately after it. The Poisson exit shows up in two places: it raises the effective discount rate from $\rho$ to $\rho + \lambda$, and it adds a flow continuation term weighted by $\lambda$. Applying standard optimal-control techniques yields a modified Euler equation in the high-growth regime

$$\frac{\dot{c}_s}{c_s} = \frac{r - \rho}{\eta} + \frac{\lambda}{\eta}\left\{\mathbb{E}_{\varphi}\left[\left(\frac{\varphi y_s + \kappa b_s}{c_s}\right)^{-\eta}\right] - 1\right\}. \tag{9}$$

This Euler equation is key. Write $c^{+} \equiv \varphi y_s + \kappa b_s$ for the consumption a household would jump to if its type arrived at $s$. Relative to the balanced-growth rate $\frac{r-\rho}{\eta} = g$, consumption growth in the high-growth spell is adjusted by $\frac{\lambda}{\eta}\{\mathbb{E}_{\varphi}[(c^{+}/c_s)^{-\eta}] - 1\}$, the expected jump in marginal utility at the draw. This single term carries both the consumption smoothing force due to elevated income growth and the precautionary motive induced by uncertainty about $\varphi$. Before turning to the implications for excess savings that follow from this Euler equation, I solve the rural household problem.

**Rural household problem.**

Note that the value of moving to the urban sector always looks the same, as households undergo the same expected transition, up to the constant growth term $g$. I thus can write this urban expected value in normalized form, $V_t^{\mathrm{U}} = v^{\mathrm{U}} e^{(1-\eta)gt}$. The migration margin then delivers the following result.

**Proposition 1** (Migration Indifference Condition)**.** *In any equilibrium with ongoing rural–urban migration, the migration condition holds with equality,* $V_t^{R} = \tau^{\eta-1} V_t^{U}$.

*Proof.* By contradiction. If $\tau^{\eta-1}V_t^{\mathrm{U}} > V_t^{\mathrm{R}}$, a positive mass of households would instantaneously move to the urban sector until the value of moving and staying is equalized. If $\tau^{\eta-1}V_t^{\mathrm{U}} < V_t^{\mathrm{R}}$, households would stay in the rural sector, but since productivity growth makes the urban sector more attractive, there would eventually be a point when the two values are equal, and from then on households are indifferent between staying and migrating, which is sustained by a continuous rural outflow that induces wage growth in the rural sector in lock-step with productivity growth in the urban sector. □

The value of moving to the urban sector equals the value of remaining rural despite the higher urban wage, since the wage gap compensates for the utility cost of migration and for the risk in the urban sector. Note that growth generated by reallocation across sectors therefore does not represent an unexploited increase in expected lifetime income against which households want to borrow.

The value of staying in the rural sector thus equals

$$V_t^{\mathrm{R}} = \frac{\left(w_t^{\mathrm{R}}\right)^{1-\eta}}{(1-\eta)\,\kappa}. \tag{10}$$

The rural outflow necessary that keeps households indifferent between staying or migrating simply follows from finding the decline in the rural share that delivers wage growth at rate $g$ given by

$$\frac{\dot{L}_t^{\mathrm{R}}}{L_t^{\mathrm{R}}} = -\frac{g}{1-\chi}. \tag{11}$$

I define the rural–urban wage gap as urban compensation relative to rural compensation for a household at the point of entry, before it has accumulated any urban human capital, $\omega_t \equiv w_t^{\mathrm{U}}/w_t^{\mathrm{R}} = A_t^{\mathrm{U}}/w_t^{\mathrm{R}}$. The rural outflow in (11) is exactly the one that makes $w_t^{\mathrm{R}}$ grow at rate $g$, and $A_t^{\mathrm{U}}$ grows at $g$ as well, so the wage gap is constant along the transition path.

Lastly, note that I can match any rural share at time zero, and any rural-urban wage gap, by using the free parameters $\tau$ and $A^{\mathrm{R}}$.[8] Figure 4 sketches out the evolution of income for a random household that migrates from rural to urban, which summarizes the key elements of the model.

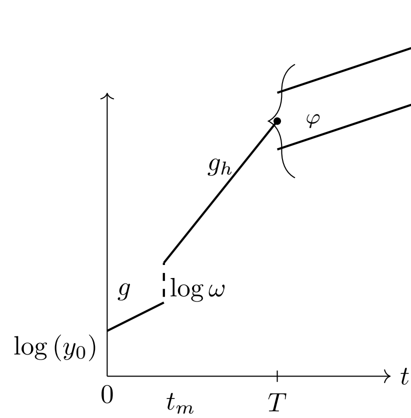


Figure 4. Income process from household starting in the agricultural sector

I next analyze whether this model features capital inflows or outflows along the transition path. I start by showing how the structure weakens the consumption-smoothing motive, and end with a condition that precisely determines whether there are capital outflows along the transition path.

### 3.3 Decoupling Aggregate Growth and Consumption Smoothing

In canonical incomplete-markets models, households face idiosyncratic shocks around a common aggregate growth trend and therefore still expect to become substantially richer as the economy converges. For economies that quickly catch-up in terms of income with the rest of the world, this dominates precautionary savings. The model here departs from this benchmark along two dimensions, each of which raises

[8] Finding the right parameters clearly depends on the value of $v^{\mathrm{U}}$, but for the purposes of the paper, it does not matter what these are, since $L_0^{\mathrm{R}}$ and $\frac{w^{\mathrm{U}}}{w^{\mathrm{R}}}$ are directly taken from the data.

aggregate growth without raising, one for one, the growth that an individual household expects.

**Uneven and Risky Growth.** Consider the income gain a household expects upon entering the urban sector, net of the wage gap and relative to the aggregate trend. Its income grows at $g_h - g$ for a random time $T \sim \text{Exp}(\lambda)$ and then draws $\varphi$, so the long-run multiplier on its income is $m = e^{(g_h-g)T}\varphi$. Because the Poisson exit makes $e^{(g_h-g)T}$ Pareto distributed, the expected multiplier is

$$\mathbb{E}[m] = \frac{\lambda}{\lambda - (g_h - g)}, \tag{12}$$

which requires $\lambda > g_h - g$ and equals $3.5$ under the baseline calibration. This is what every household would receive if catch-up growth within the urban sector were identical across households. It is not how a risk-averse household values the uneven and risky urban growth experience. The certainty equivalent of the lottery $m$ for a household with relative risk aversion $\eta$ is $m^{\text{CE}} = \left(\mathbb{E}[m^{1-\eta}]\right)^{1/(1-\eta)}$, and setting aside the type draw for the moment ($\varphi \equiv 1$), for $\eta = 2$ it equals[9]

$$m^{\text{CE}} = \frac{\lambda + (g_h - g)}{\lambda} = 1.71. \tag{13}$$

A household that will on average see its income rise by a factor of $3.5$ values the prospect as if it were a certain rise by a factor of $1.7$. The gap highlights how risky and uneven growth substantially reduces consumption smoothing pressure. Aggregate growth is disproportionately driven by households in the emerging upper tail of the distribution, those who happen to remain in the high-growth regime for a long time. From any individual household's point of view, landing in that tail is an unlikely outcome that is heavily discounted by the curvature of the utility function.[10]

**Reallocation across Sectors and Rural-Urban Wage Gaps.** The second channel operates through the migration margin and requires no risk at all. Proposition 1 established that in equilibrium the value of moving to the urban sector equals the value of remaining rural despite the higher urban wage, since the wage gap compensates for the cost of migration and for the risk assumed on arrival. Reallocation across sectors therefore raises measured aggregate income—it is the second term of the growth decomposition in Section 4—while creating no consumption-smoothing pressure: growth generated by reallocation does not represent an unexploited increase in expected lifetime income against which households want to borrow.

**Putting the two together.** Under the baseline calibration, aggregate income per capita rises by a factor of $4.9$ along the transition. Of this, a factor of $1.6$ is reallocation across sectors and a factor of $3.1$ is uneven growth within the urban sector—$3.5$ per migrant, less in aggregate because part of the initial urban population does not pass through the high-growth regime. A representative agent would borrow against all of it. Here the first factor generates no smoothing pressure at all, by Proposition 1, and the second is valued by a household entering the urban sector as a certain gain of $71$ percent rather than the $250$ percent it delivers on average. Aggregate income per capita rises by $390$ percent along the transition; the gain a household actually plans around is a fraction of that.

This alone, however, cannot explain excess saving. Weakening the consumption-smoothing motive reduces the desire to borrow, but it cannot by itself produce saving. That requires households to face the

[9]This is a static comparison: it treats the long-run multiplier as a one-shot lottery and abstracts from discounting and from the timing of the draw, in order to isolate the size of the growth miracle and the effect of its unevenness. The household problem in Section 3.4 and the quantitative analysis in Section 4 account fully for discounting at $r$ and for the time path of income.

[10]With log utility the comparison is between the expected log gain $(g_h - g)/\lambda = 0.71$ and the log of the expected gain $\log(3.5) = 1.25$; the result is the same, and it survives in the limit $\lambda \downarrow g_h - g$, where $\mathbb{E}[m]$ diverges while $m^{\text{CE}}$ stays finite. One can thus construct an arbitrarily large growth miracle whose effect on any household's expected utility is modest.

possibility of ending up worse off, which is the subject of the next subsection.

### 3.4 Capital In- or Outflows

Write the Euler equation in the high-growth regime in terms of post-draw consumption. With $\kappa \equiv r - g$ and $c^+ \equiv \varphi y + \kappa b$ the consumption a household would jump to if its type arrived now, equation (9) reads

$$\frac{\dot{c}}{c} = g + \frac{\lambda}{\eta}\Big\{\mathbb{E}_\varphi\big[(c^+/c)^{-\eta}\big] - 1\Big\}. \tag{14}$$

Consumption grows at the balanced rate $g$, adjusted by $\lambda/\eta$ times the expected jump in marginal utility at the draw. Whether the household saves or borrows in the high-growth regime is read off the sign of the bracket.[11]

**Proposition 2** (Uneven growth alone does not generate excess saving)**.** *Suppose there is no type risk, $\varphi_i = 1$ for all $i$, so that the only uncertainty a household faces is the duration of its high-growth spell. Then households borrow along the entire transition path and the economy runs current account deficits.*

*Proof.* Appendix 6. □

Uncertainty about how much better off a household will become does not generate the required precautionary motive. With $\varphi \equiv 1$ the bracket in (14) is negative at every point in the high-growth regime: whatever happens, the household will be at least as well off after the draw as before it, so the expected marginal utility of consumption tomorrow is lower than today's, and the household consumes ahead of income.[12] Uneven growth therefore does one important job—it weakens the desire to borrow—but it cannot by itself reverse the sign. Duration risk is upside risk, and upside risk induces borrowing. To generate excess saving, households must face the possibility of being worse off, which the random growth process doesn't deliver: it only governs the length of the growth spell, and can never leave a household worse off than where they started.

**Proposition 3** (Excess saving along the transition path)**.** *Let $\mathbb{E}[\varphi] = 1$ and $g_h > r$. Households in the high-growth regime accumulate assets, and the economy runs current account surpluses along the transition path, if and only if*

$$\frac{g_h - g}{\lambda}\,\eta \;<\; \mathbb{E}_\varphi\big[\varphi^{-\eta}\big] - 1. \tag{15}$$

*Proof.* Evaluate (14) at the point of entry. A household migrates with $b = 0$; if it consumed its income then $c = y$ and $c^+/c = \varphi$, so

$$\frac{\dot{c}}{c} = g + \frac{\lambda}{\eta}\Big\{\mathbb{E}_\varphi\big[\varphi^{-\eta}\big] - 1\Big\}.$$

Absent risk the bracket vanishes and consumption grows at the balanced rate $g$, slower than income by exactly $g_h - g$: the household borrows against future income. Risk adds the term $\frac{\lambda}{\eta}\{\mathbb{E}_\varphi[\varphi^{-\eta}] - 1\}$, which is strictly positive by Jensen's inequality for any non-degenerate mean-preserving $F$, and the household

[11] Borrowing is bounded. Let $\underline{\varphi}$ be the lower bound of the support of $\varphi$. Feasibility requires $\kappa b_t/y_t > -\underline{\varphi}$: otherwise a draw of $\underline{\varphi}$ leaves the household with non-positive total wealth, violating the no-Ponzi condition. For the lognormal draw used below, $\underline{\varphi} = 0$ and households never borrow; even an arbitrarily small amount of downside risk suffices to rule out borrowing against future income. This does not by itself generate outflows, which require positive accumulation.

[12] The counterfactual is not marginal. Under the baseline calibration, a household in the high-growth regime settles at a consumption level 17 percent above its income and sustains a debt of more than eight years' income. Appendix 6 derives these values in closed form.

accumulates assets precisely when it exceeds $g_h - g$, which is (15). A household whose optimal consumption path is steeper than its income path must, to satisfy its budget constraint, begin below income and accumulate assets; the reverse holds for households with a flat consumption path relative to income. $\square$

The left-hand side of (15) is the consumption-smoothing force: expected log income growth in the high-growth regime, $(g_h - g)/\lambda$, scaled by risk aversion. The right-hand side is the precautionary force, the expected marginal-utility gain from the permanent draw, which by Jensen's inequality is strictly positive for any non-degenerate mean-preserving distribution. Excess saving arises when the second dominates the first. The condition makes the two channels of the previous subsection visible. Uneven growth enters through $\lambda$: a shorter expected spell shrinks the left-hand side without touching the right. The wage gap does not enter the condition at all: by the migration indifference condition, growth generated through reallocation creates no consumption-smoothing pressure.

Appendix 6 shows that (15) is necessary and sufficient for the stationary asset-to-income ratio in the high-growth regime to be positive, that this steady state is saddle-path stable, and that $b/y$ rises monotonically along the path from $b = 0$.

**How much risk is enough.** For the lognormal type draw, $\mathbb{E}[\varphi^{-\eta}] = \exp\big(\sigma^2\eta(1+\eta)/2\big)$, and (15) becomes

$$\sigma^2 \;>\; \frac{2}{\eta(1+\eta)} \log\Big(1 + \eta\,\frac{g_h - g}{\lambda}\Big). \tag{16}$$

Under the baseline calibration, excess saving requires a variance of only $\sigma^2 = 0.30$. The calibrated value is $0.49$, disciplined by long-run income dispersion rather than by saving or capital flows, and therefore comfortably exceeds this threshold. The quantitative importance of uneven growth becomes clear by comparing this result with an economy in which the same aggregate growth miracle is distributed evenly across households. Holding aggregate convergence fixed but letting every household grow at a common rate before drawing the same permanent type, the variance required to generate excess saving rises to $\sigma^2 = 4.75$.[13] Uneven growth therefore reduces the amount of permanent income risk required to overturn consumption smoothing by a factor of roughly sixteen. Both economies contain the same form of permanent downside risk; what differs is whether catch-up growth itself is distributed evenly or unevenly across households.

There exists no closed-form solution for the transitional dynamics of consumption and assets within the high-growth phase of a household. It is possible, however, to characterize the qualitative properties of the non-linear system using the normalized Euler equation, similar to the analysis of the neoclassical growth model in continuous time; Appendix 6 provides the phase diagram. When (15) holds, household consumption and assets grow at a rate higher than income initially and converge from above to the growth rate of income in the high-growth regime, so that the consumption-to-income and asset-to-income ratios approach constants.

### 3.5 The Role of the Rural–Urban Transition

The two channels of Section 3.3 both show up in the aggregate growth rate. Although households draw types and movers see their wage jump, this randomness washes out in the aggregate, and per capita growth

[13] Appendix 6.3 sets out the calculation. Household income inequality in the United States corresponds to a log variance close to one and labor income inequality to roughly $0.6$ (Krueger, Mitman, and Perri, 2016).

simply reads

$$g_{agg}(t) = \underbrace{(g_h - g)\frac{Y_{0,t}}{Y_t} + \left(\frac{g}{1-\chi}\right)\frac{(\omega-1)L_t^{\mathrm{R}}}{Y_t}}_{\text{catch-up growth}} + \underbrace{g}_{\text{long-run growth}}, \tag{17}$$

where $Y_{0,t}$ is the income of households still in the high-growth regime. Catch-up growth has two sources. The first term is uneven growth within the urban sector, weighted by the income share of households that are still growing fast. The second is reallocation: households leave agriculture at rate $g/(1-\chi)$ and each mover gains the wage gap $\omega - 1$.

This decomposition shows how the dual-sector structure helps chip away at consumption smoothing pressure. A one-sector version of the model would have to generate the entire growth miracle through the first term. Matching the same aggregate convergence would then require faster or more persistent household-level catch-up growth, raising $(g_h - g)/\lambda$, precisely the consumption-smoothing force on the left-hand side of the outflow condition (15). Reallocation instead contributes to aggregate convergence without tightening this condition, because by Proposition 1 the households doing the moving are indifferent at the margin. Structural change therefore makes it possible to generate a large aggregate growth miracle without requiring households to expect correspondingly large future income gains.

The rural–urban margin also provides a natural economic interpretation for rising risks as countries grow rich. Returns to specialized human capital become particularly important as households leave traditional agriculture and enter modern urban production, where ex ante similar workers can experience very different long-run outcomes. This interpretation is consistent with the household evidence in Section 2.2: urban households hold substantially more assets relative to income than rural households, and they continue to accumulate faster over the panel. These patterns are consistent with a stronger precautionary motive on the urban side of the transition.

The income process is deliberately stylized to separate uneven growth from downside risk while preserving the analytical solution. Its particular Poisson and permanent-draw structure is not essential to the economic argument. What is essential is that uncertainty about long-run urban earnings remains unresolved when households enter the urban sector and includes outcomes in which households can end up worse off. Sources of persistent downside risk—such as unemployment, early retirement, or uncertainty about the earnings of future household members—could provide a particular microfoundation. The framework shows that any of these must generate downside risk in the sense that households can be worse off despite fast aggregate growth.

Finally, structural change is crucial to generate realistic transition dynamics. New migrants enter the high-growth and risky phase of their urban careers while earlier cohorts leave it, so the share of households exposed to unresolved risk rises and eventually falls. Section 4 shows that this compositional force generates the hump-shaped path of aggregate saving: without the continuous inflow of new households from the rural sector, the precautionary saving response would instead peak immediately and decline.

# 4 Calibration

I next quantify the model to solve for the aggregate trajectory of the economy along the transition path. The goal of this calibration is to show that the elements introduced in the model can give rise to realistic "miracle growth dynamics". As a starting point, I set $g_h = 7\%$, $g = 2\%$, and $\lambda = \frac{7}{100}$ which implies an average time spent in the high-growth regime for each household of a bit less than 15 years. Expected income growth upon migration, after netting out the effect of the urban-rural wage gap, equals

$$\begin{aligned}\mathbb{E}_i\left[\exp\left([g_h-g]T_i\right)\varphi_i\right] &= \mathbb{E}_i\left[\varphi_i\right]\int_1^\infty \frac{\lambda}{g_h-g}y^{-\frac{\lambda}{g_h-g}}dy \\ &= 1+\frac{g_h-g}{\lambda-(g_h-g)}\end{aligned}$$

where I use both the independence of the type draw, as well as the fact that the Poisson process leads to Pareto-distributed income due to catch-up growth. For the parameters picked, this amounts to convergence growth of 250%, which is multiple times larger than the growth miracle in Buera and Shin (2017). With the urban-rural wage gap set to $\omega = 2$, aggregate income per capita rises by a factor of $4.9$, or $390$ percent: the base year already contains the wage gap, so that $Y_0 = 0.75 + 0.25 \times 2 = 1.25$ while $Y_\infty = 2 \times 3.0625 = 6.125$.[14]

I set the coefficient of relative risk aversion to $\eta = 2$. I model the inequality shock as a draw from a Log-Normal distribution, i.e. $log(\varphi) \sim N(-\frac{\sigma^2}{2}, \sigma^2)$. This particular representation ensures that $\mathbb{E}[\varphi] = 1\,\forall\sigma$, so as to isolate the effect of higher inequality due to a larger variance from first-order effects that would otherwise shift up the mean and obscure analysis.[15] In order to calibrate the variance of the log of the type draw, I target the level of household inequality in the United States, implicitly assuming that the miracle economy converges to this long-run equilibrium. For a household that passes through the high-growth phase, the limiting variance of log income is $\left(\frac{g_h-g}{\lambda}\right)^2 + \sigma^2$, and $\sigma^2 = .49$ delivers a value close to one. This is consistent with measures of household income inequality in the US (Krueger, Mitman, and Perri, 2016).[16] The limiting variance in the urban population as a whole is lower, $0.88$, because the share $M_{1,0}$ already urban at time zero never passes through the high-growth phase. This is a composition effect rather than a statement about the risk any individual household faces. For the dispersion of the initial urban stock I set $\sigma_0^2 = 0.10$. The CHIP data begin in 1988 and I am not aware of a direct measure of urban income dispersion in China before the reforms, so this value is provisional; it does not affect the saving or capital-flow paths, which are invariant to it.

[14] The GDP gains are larger than the welfare-gains of the growth miracle for two reasons. First, we would have to be properly discount future growth that only materializes far in the future. Second, there is a distributional cost of the growth miracle. Ex ante, the household risk embodied in the growth miracle leads to an even higher effective discount factor. The actual growth miracle is also slightly smaller because of the agents $M_{1,0}$ in the city that do not experience miracle growth. That is, the per capita growth rate after netting out the urban-rural wage gap would be roughly 206% instead of 250%.

[15] Log normality is a common assumption in the context of cross sectional wage distributions, albeit not innocuous (Guvenen et al., 2015). Note that due to uneven growth the right tail of the log of income will be dominated by the exponential distribution.

[16] Actually, the relevant statistic here is provided by De Magalhães and Santaeulàlia-Llopis (2018) who themselves rely on unpublished data by Krueger, Mitman, and Perri (2016). Alternatively, recent work by Guvenen et al. (2017) measures the variance of the log of income from tax returns around .8.

| parameter | baseline value |
|---|---|
| discount factor | $\rho = .01$ |
| coefficient of relative risk aversion | $\eta = 2$ |
| log-variance of type draw | $\sigma^2 = .49$ |
| Poisson arrival rate | $\lambda = 0.07$ |
| industrialized growth | $g = 2\%$ |
| miracle growth | $g_h = 7\%$ |
| urban-rural wage gap | $\omega = 2$ |
| elasticity of agr. output with respect to labor | $\chi = \frac{5}{9}$ |
| initial agr. share | $L_0^{\mathrm{R}} = 75\%$ |
| initial share of agents that know their type | $M_{1,0} = 17.5\%$ |
| log-variance of the initial urban stock | $\sigma_0^2 = 0.10$ |

Table 3. Baseline calibration.

Recall from Section 3 that $\omega$ denotes the wage gap between urban and rural households before the urban household has accumulated any urban human capital, $\omega = w_{t+\Delta}^{\mathrm{U}}/w_t^{\mathrm{R}}$. This wage gap is going to be another source of convergence. I set $\omega = 2$, so that a household entering the urban sector earns twice the rural wage, which I view as a lower bound. Fan and Zou (2019) suggest that the wage of an unskilled urban worker is three times that of a rural worker in China, and they provide evidence that this ratio is relatively stable. The agricultural share is set high at 75% which leads to powerful catch-up growth. Note that I assume that an initial share of households already has learned their type and is in group $M_{1,0}$. If I assumed that all agents that are in the city at time zero started growing fast, then this mass point would dominate the dynamics of aggregate savings completely.[17]

Lastly, I need to set the parameter $\chi$ which determines the curvature on the rural production function. This is a key parameter as it governs the speed at which households move out of the agricultural sector. The next subsection shows how to estimate $\chi$ through the lens of the model. Recall that $L_t^{\mathrm{R}} = L_0^{\mathrm{R}} \exp\left(-\frac{g}{1-\chi} t\right)$ describes the evolution of the rural share. This law of motion of agricultural employment leads to the following estimating equation,

$$\log\left(L_{t,c}^{\mathrm{R}}\right) = \beta_{0,c} + \beta_1 * t + \epsilon_{t,c}, \tag{18}$$

where I added a random error term. The estimating equation (18) can efficiently be estimated using a random effects model across a sample of miracle economies. In doing so, I acknowledge that different initial conditions lead to different initial agricultural shares while maintaining that the technology coefficient $\chi$ is constant across economies. I estimate it on a sample of five miracle economies (Germany, Taiwan, Japan, Korea, and China), each beginning from the point in time when it started to reform, following Buera and Shin (2013). Maintaining that $g$ is equal to 2 %, consistent with long-run growth in developed economies over the twentieth century (Lucas, 2018), implies an estimate of $\hat{\chi} \approx \frac{5}{9}$.[18] Figure 5 shows the fit of model-implied structural change relative to the observed agricultural employment. The fit is nearly perfect for all economies but China. One wonders whether this is a manifestation of the detrimental effects of the Hukou

[17] An unpleasant side effect of that is that the aggregate saving rate would not deliver a hump-shaped pattern. Instead, it would mimic the individual saving rate, first shooting up and then monotonically declining.

[18] Note that the sample of countries is too small for the fixed estimator to be consistent based on cross sectional variation. Nevertheless, the results of the random and fixed effects model are very similar. The fit of the model in a R-squared sense is excellent, and accounts for more than 95% of variation in the data.

system potentially hampering the process of structural change, as discussed in Tombe and Zhu (2019).[19]

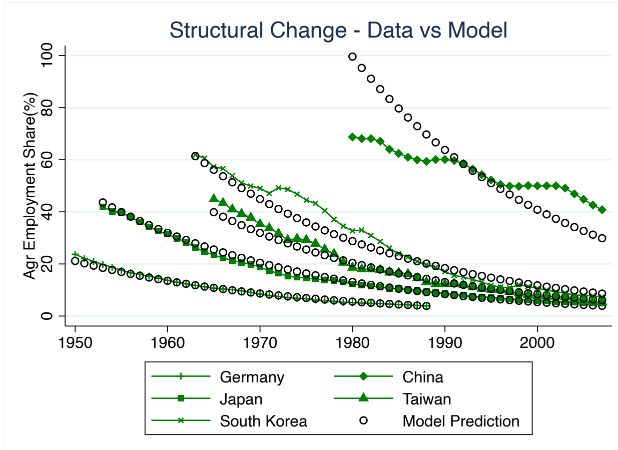


Figure 5. Prediction based on random effects model.

### 4.1 Income Growth

Since labor is normalized to one and constant, the aggregate growth rate in (17) is also the per capita growth rate. Figure 6 plots the aggregate growth rate for the parameter values chosen in the calibration. The growth miracle is sizable, and comparable to the experience of Taiwan. Note that the aggregate growth rate can be larger than the growth rate measured in the micro data, consistent with findings in the literature (Santaeulalia-Llopis and Zheng, 2018). The canonical income process fails to capture this. In the context at hand, it arises for two reasons. First, note that the urban-rural wage gap contributes to higher aggregate growth. It is likely that this income jump is missed in the micro data, or even discarded on purpose as an outlier. Second, fast-growing households achieve a relatively higher share in aggregate GDP in the long run, thus dominating aggregate dynamics and raising the growth rate relative to the simple average in the micro data. Figure 6 also shows the convergence in terms of log output per capita relative to the United States.

### 4.2 Income Inequality

The model delivers closed form solutions for the distribution of income along the transition path. The derivation can be found in the appendix in subsection 6.2. It is well known that heterogeneous growth rates give rise to a fat-tailed income distribution (Luttmer, 2011; Gabaix et al., 2016; Aoki and Nirei, 2017). The stationary distribution has a Pareto right tail with index $\alpha = \frac{\lambda}{g_h - g}$, so that $P(y > Y) \sim Y^{-\alpha}$; under the baseline calibration $\alpha = 1.4$. Finite aggregate convergence requires $\alpha > 1$, which is the same condition as $\lambda > g_h - g$.

Figure 7 shows the CDF of the log of normalized income for different decades. Importantly, while the distribution fans out overall, more and more weight is being shifted to the right tail that is composed of households that remained in the high growth regime for a relatively long time. When every household passes through the high-growth phase, the normalized stationary distribution of the log of income is exactly the exponentially modified Gaussian, the sum of two independent random variables, one of which is

[19] The reader might wonder whether risk in the urban sectors adds to the rural-urban wage gap in the form of a compensating differential: this is indeed the case as I show in the appendix in subsection 6.1.1.

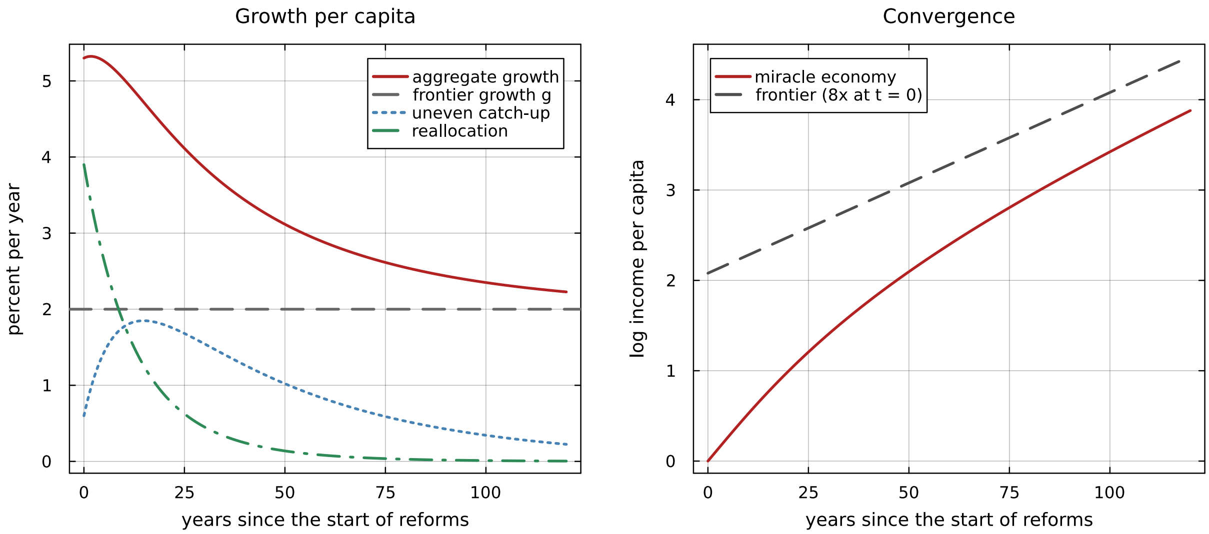


Figure 6. Per capita growth rates and log output computed using the values provided in table 3. Per capita GDP is assumed to be 8 times larger in the US at time zero.

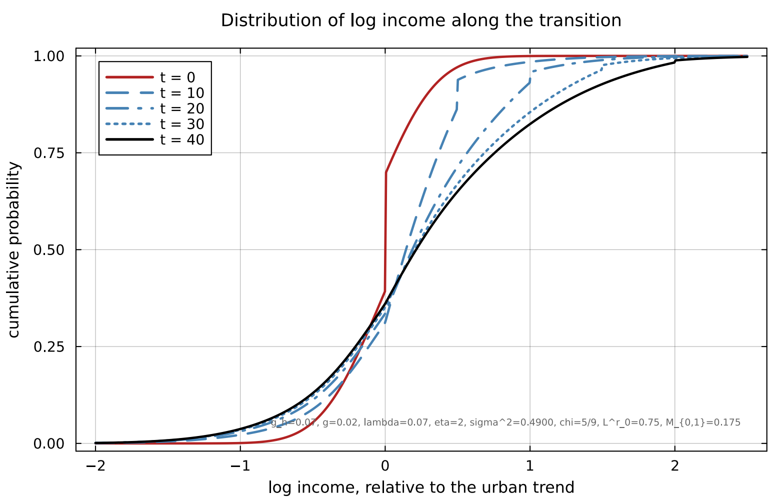


Figure 7. CDF of log income among urban households, net of the urban trend, at $0$, $10$, $20$, $30$ and $40$ years after the reforms start. Later periods lie to the right. The visible step is the mass of households already in the high-growth regime at time zero, all of whom share the same accumulated growth $(g_h - g)\,t$; it shrinks exponentially as those households draw their type.

normal (what I call the type draw) and one of which is exponentially distributed (time spent in the high-growth regime, scaled by the growth rates). With a positive initial urban stock the limiting distribution is a mixture of this object and the distribution of that stock. The departure is modest but not zero: the maximum deviation in probability is $0.06$ when the initial stock carries the same dispersion as the type draw, and $0.12$ when it carries none. Armed with this CDF I can compute the log variance of income for non-agricultural households displayed in table 4. The results are broadly in line with the fast rise of inequality in China. The 1980 row is pinned by the initial condition $\sigma_0^2$ rather than being a prediction; the four rows below it are. The model tracks 1988, 2002 and 2013 closely and understates 1995, a year in which the CHIP series itself is non-monotone.

| year | data | model |
|---|---|---|
| 1980 | $NA$ | 0.07 |
| 1988 | $0.17$ | 0.16 |
| 1995 | $0.37$ | 0.26 |
| 2002 | $0.36$ | 0.36 |
| 2013 | $0.5$ | 0.5 |

Table 4. The data moments come from the CHIP and concern non-agricultural occupations for household heads between 23 till 60 years of age, smallest 2 percent of income realizations dropped. Variance of log of income of the model in the last column.

### 4.3 Capital Outflows

Finally, I compute the capital flows of the economy along the transition path. To do so, I need to first solve for the transitional consumption and asset accumulation dynamics in the high-growth regime. This is simple, however, since the problem of the household in the high growth regime always looks the same (up to some linear scaling factor). Intuitively, households always go through the same dynamics, albeit at different starting wages $w_t^{\mathrm{U}}$. All choice variables are then simply scaled by income but otherwise unchanged.

Figure 8 shows the evolution of asset-to-income and consumption-to-income ratio, consistent with the standard bufferstock-saving model (Carroll, 1997). Time $0$ here stands in for the time the household entered the urban sector, and households follow these dynamics as long as they are in the high growth regime. Again, note that precautionary savings on its own cannot generate the non-monotone transition dynamics we see in the data, as the plot shows that for each household, saving rates are monotonically rising or falling, depending on whether the equilibrium features inflows or outflows.

Going back to the empirical exercise in section 2, the asset-to-income ratio is informative about the precautionary motive, at least through the lens of the model. A larger variance of the type draw leads to a larger asset-to-income ratio. High urban asset-to-income ratios, then, are indicative of high human capital risk in modern production. The parameterization already reveals that the growth miracle is going to be accompanied by capital outflows, since households accumulate assets in the high growth regime. This is not guaranteed, and for more "even" growth miracles this would not be the case.

In a final step, I compute aggregate saving rate along the transition path, which coincides with the current account. Figure 9 plots the aggregate savings rate.

A success of the model is that it can replicate the hump-shaped saving rate that is characteristic of growth miracles. The current account surplus peaks at $7.1$ percent of GDP some eighteen years after the reforms begin and declines slowly thereafter, settling at $3.7$ percent in the very long run. The magnitude is of the same order as the Chinese experience: Figure 2 shows the Chinese current account peaking at seven percent of GDP in 2007. Note that no saving, current account, or asset moment enters the calibration, so the closeness of the two peaks is a surprising success of the framework.

The right panel of Figure 9 shows the same growth episode in a standard representative-agent economy, where growth is evenly shared and there is no risk. The household has perfect foresight over the entire catch-up path and discounts it at the world interest rate $r$, so its consumption is pinned by the intertemporal budget constraint at $c_0 = \kappa \int_0^\infty e^{-rs} Y_s \, ds$. It consumes $2.2$ times its income at $t = 0$ and runs a current account deficit of $120$ percent of GDP. This is the strength of the consumption-smoothing force the mechanism has to overturn, and it is roughly thirty times the magnitude of the flows the model generates. The benchmark model also fails in terms of transition dynamics, which are monotone, and generically rule out hump-shaped savings rates.

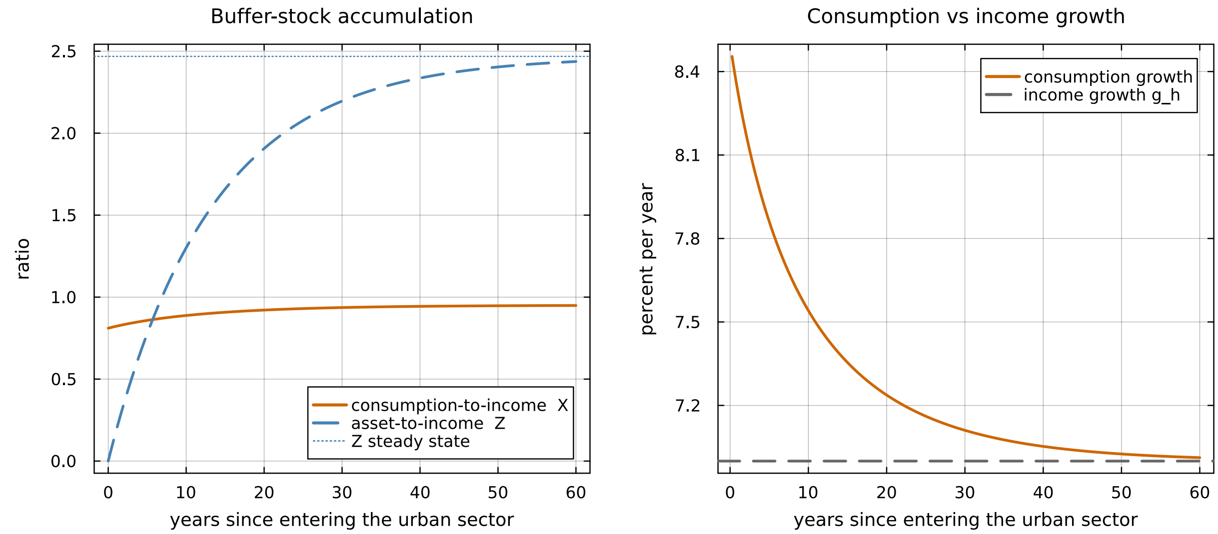


Figure 8. Transitional dynamics in the high-growth regime, using the parameters in table 3. Time is measured from the date the household entered the urban sector. The left panel shows the consumption-to-income and asset-to-income ratios, with the steady-state asset ratio marked; the right panel shows consumption growth against income growth $g_h$, which is the buffer-stock pattern of Proposition 3: consumption grows faster than income at first and converges to it from above.

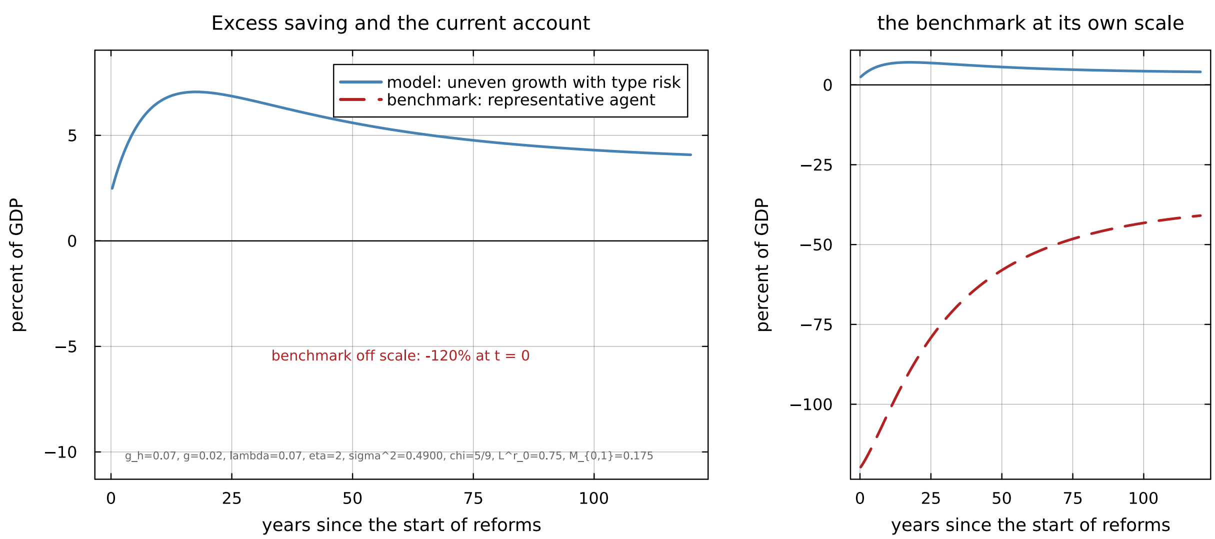


Figure 9. Excess saving and the current account along the transition path. Since there is no capital in the model, saving in excess of investment coincides with the current account of the small open economy. The left panel is scaled to the model; the representative-agent benchmark runs off the bottom of it and is shown at its own scale on the right.

Figure 10 shows the flow together with the stock it accumulates. Net foreign assets rise to 183 percent of GDP and stay there. That is a large number and it is worth being clear about where it comes from: the interest rate is fixed by assumption, so once the transition is complete nothing in the model closes the external position. This is a property of the small open economy rather than of the mechanism. An endogenous world interest rate, or a country premium rising in the net foreign asset position, would damp the stock without changing the sign or the timing of the flow.

To understand why the model delivers a hump-shaped saving rate it is helpful to look at the movements

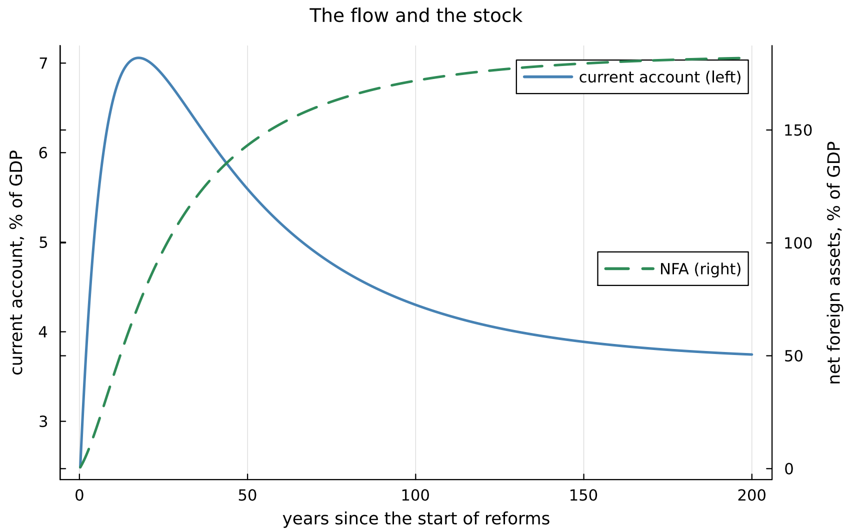


Figure 10. The current account and the net foreign asset position it accumulates. The world interest rate is fixed, so nothing in the model closes the external position once the transition is complete.

from households out of agriculture, and from the high growth to the low growth regime. Figure 11 shows the declining share of agriculture. The hump-shaped saving rate can only emerge as a compositional effect. That is, there needs to be an increasing share of precautionary savers relative to total output for some time. This is precisely what happens as figure 11 shows. The share of agents in the fast-growth regime is hump-shaped, and the aggregate saving rate can inherit those dynamics. Whether this is the case depends on the values for $g, \chi$ and $\lambda$, which govern inflow and outflow into $M_{0,t}$ and $M_{1,t}$.

Figure 9 shows that this model can solve the capital flow puzzle for the right parameter values. The heterogeneity in the growth process and rural-urban differences in human capital risk are key deviations from the representative-agent neoclassical model that make this feasible.

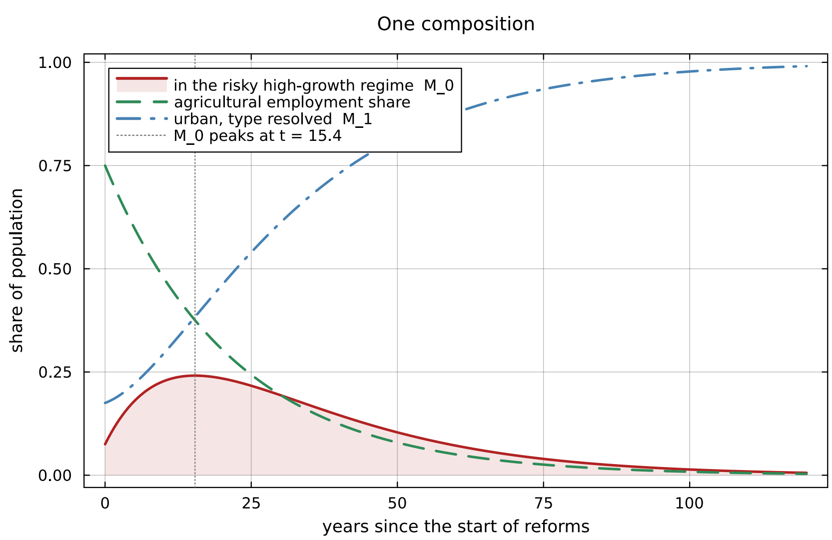


Figure 11. Changing share of population in agriculture, and in the high-growth regime.

## 5 Conclusion

Fast-growing emerging economies save in excess of domestic investment and export capital, precisely when consumption smoothing predicts that they should borrow. I have argued that the resolution lies in the uneven nature of miracle growth. When catch-up growth is concentrated in an emerging upper tail rather than distributed evenly across households, aggregate growth substantially overstates the growth that individual households expect when making consumption and saving decisions. Uneven growth therefore weakens the force that makes the puzzle hard in the first place: the expectation of very high future household income.

Uneven growth alone, however, is not sufficient. Uncertainty about how much better off a household will become weakens the consumption smoothing motive but is not sufficient to generate savings. Households must also face the possibility of ending up worse off. The two ingredients therefore play distinct roles: uneven growth weakens consumption smoothing, while permanent downside risk generates precautionary saving. Rural–urban structural change further strengthens the mechanism by generating aggregate growth through reallocation without corresponding consumption-smoothing pressure, while the continuous inflow of migrants generates realistic transition dynamics.

The tractability of the model makes these forces transparent. The transition of the income distribution and the condition determining capital flows can be characterized in closed form, allowing the mechanism to be traced directly to its primitives. Quantitatively, uneven growth reduces the log variance required to generate excess saving from $4.75$, the requirement when the same aggregate convergence is distributed evenly across households, to $0.30$, a factor of roughly sixteen. The model reproduces well the evolution of income inequality in China, and generates a hump-shaped path of excess savings, with a current account surplus peaking at around seven percent of GDP.

The model is deliberately parsimonious. Motivated by Gourinchas and Jeanne (2013), I focus on excess saving and abstract from physical capital in the baseline. Likewise, the particular income process is chosen for tractability; the mechanism requires only that uncertainty about long-run urban earnings remains unresolved upon entering the urban sector and includes the possibility of adverse long-run outcomes. Companion work on South Africa (Trouvain, 2026) provides direct evidence on these features of the rural–urban transition.

The broader point extends beyond this particular application. Structural change matters not only because it reallocates workers across sectors and raises aggregate productivity. When households face different income processes and risks across sectors, the process of reallocation also shapes aggregate saving and other macroeconomic dynamics along the transition path. This paper provides a first step toward bringing this dimension of structural change into heterogeneous-agent macroeconomics.

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## 6 Theory Appendix

### 6.1 Simple Infinite Horizon Economy with Poisson Arrival of Type

**Deriving equation (8):**
I derive the relationship for a general utility function $u$. Throughout, $V^{\mathrm{U}}(\cdot)$ denotes the post-draw value function in (7).

$$\begin{aligned}
V_{t_m} &= \max \mathbb{E}_{\hat{t}} \mathbb{E}_{\varphi} \left[ \int_{t_m}^{\infty} \exp\left(-\rho[s-t_m]\right) u\left(c_s\right) ds | T = \hat{t} \right] \\
&= \max \mathbb{E}_{\hat{t}} \left[ \int_{t_m}^{\hat{t}} \exp\left(-\rho[s-t_m]\right) u\left(c_s\right) ds + \exp\left(-\rho[\hat{t}-t_m]\right) \mathbb{E}_{\varphi} V^{\mathrm{U}}\left(b_{\hat{t}}, \varphi y_{\hat{t}}\right) \right] \\
&= \max \int_{t_m}^{\infty} \lambda \exp\left(-\lambda[t-t_m]\right) \left[ \int_{t_m}^{t} \exp\left(-\rho[s-t_m]\right) u\left(c_s\right) ds + \exp\left(-\rho[t-t_m]\right) \mathbb{E}_{\varphi} V^{\mathrm{U}}\left(b_t, \varphi y_t\right) \right] dt,
\end{aligned}$$

where the first line conditions on the arrival time. The second line splits up the integral, conditional on the arrival time, into the time before and after the agent learned about their type. Note that the second line also implicitly reflects the independence of the Poisson arrival process, and the agent's type. This allows me to simply compute the expectation over the type-space.

The next step is to change the order of integration. The boundaries of integration are $t > s > t_m$. Changing the order of integration means that we first integrate over $t$. In that case, the lower boundary is $s$, and there is no upper bound on the values that $t$ can take. This gives the following solution

$$\begin{aligned}
V_{t_m} &= \max \int_{t_m}^{\infty} \exp\left(-\rho[s-t_m]\right) u\left(c_s\right) \left[ \int_{s}^{\infty} \lambda \exp\left(-\lambda[t-t_m]\right) dt \right] ds \\
&+ \int_{t_m}^{\infty} \lambda \exp\left(-\lambda[t-t_m]\right) \exp\left(-\rho[t-t_m]\right) \mathbb{E}_{\varphi} V^{\mathrm{U}}\left(b_t, \varphi y_t\right) dt \\
&= \max \int_{t_m}^{\infty} \exp\left(-\left(\lambda+\rho\right)\left[s-t_m\right]\right) \left[ u\left(c_s\right) + \lambda \mathbb{E}_{\varphi} \left[ V^{\mathrm{U}}\left(b_s, \varphi y_s\right) \right] \right] ds.
\end{aligned}$$

**Dynamics in the high-growth regime.**
To prove and understand the propositions I first derive the dynamics for households in the high growth regime. Next, I will show how this fits into the whole equilibrium and in particular into the migration decision. The dynamics are very similar to the standard neoclassical growth model, and a phase diagram analysis allows for a general characterization.

Consider first the households in the high-growth regime. Recall the household Euler equation as well as the budget constraint that determine the dynamics in the high growth regime:

$$\begin{aligned}
\frac{\dot{c}_s}{c_s} &= \frac{\lambda}{\eta} \left\{ \mathbb{E}_{\varphi} \left[ \left( \frac{c_s}{\varphi y_s + \left[g\left(\eta-1\right)+\rho\right] b_s} \right)^{\eta} - 1 \right] \right\} + \frac{r-\rho}{\eta} \\
\dot{b}_s &= r b_s + y_s - c_s
\end{aligned}$$

The first step is to obtain a system of differential equations with a steady state. In the main part of the paper I call this "pseudo" steady state. The reason is that households won't reside in this equilibrium forever, but are pulled out randomly according to the Poisson arrival process of their type. Nonetheless, I can use the steady state analysis in combination with a phase diagram to understand the equilibrium dynamics of households in the high-growth regime. The only difference is that for the actual solution of the model, I would need to send households onto the convergence process toward the pseudo steady state and then pull them out randomly consistent with the Poisson process.

Since I have growth in my model, $\dot{c_s} = 0$ is not going to be a solution. In order to obtain a stationary system, I define a new system where consumption and assets are normalized by income. This choices is motivated by Carroll (1997) who shows that asset-to-income ratios are stationary in a particular type of precautionary savings models.[20] His insight generalizes to the framework at hand as well. Let $X_s := \frac{c_s}{y_s}$ denote the consumption-to-income ratio, and let $Z_s := \frac{b_s}{y_s}$ denote the asset-to-income ratio. This leads to the following differential equations

$$\frac{\dot{X}_s}{X_s} = -(g_h - g) + \frac{\lambda}{\eta}\left\{\mathbb{E}_\varphi\left[\left(\frac{X_s}{[g(\eta-1)+\rho]\,Z_s+\varphi}\right)^\eta\right]-1\right\} \tag{19}$$

$$\frac{\dot{Z}_s}{Z_s} = -(g_h - r) + \frac{1}{Z_s}(1-X_s)\,. \tag{20}$$

The loci for equation (19) and (20) are given by

$$X_{**} = \left\{\frac{(g_h-g)}{\lambda}\eta+1\right\}^{\frac{1}{\eta}}\left\{\mathbb{E}_\varphi\left[\left(\frac{1}{\varphi+[g(\eta-1)+\rho]\,Z_{**}}\right)^\eta\right]\right\}^{-\frac{1}{\eta}} \tag{21}$$

$$Z_{**} = \frac{1-X_{**}}{g_h-r} \tag{22}$$

where $**$ denotes the steady state values. Now there are two equilibria that could emerge: a situation with precautionary savings and capital outflows, or an equilibrium with consumption smoothing and capital inflows. I am going to focus on the equilibrium with precautionary savings, which means $Z_{**} > 0$.

In that case is easy to show that (21) is increasing in $Z_{**}$ and (22) is strictly decreasing in $X_{**}$ where I assume $g_h > r$, so that the high-growth spell is a period of genuine catch-up relative to the world interest rate. Since $r = \eta g_h + \rho$, this bounds risk aversion from above: under the baseline calibration the admissible range is $\eta < (g_h - \rho)/g_h = 3$. This leads to a unique steady state solution (if it exists). For existence, we need the intercept of (21) to be below the intercept of (22) which is satisfied as long as $\left(\frac{g_h-g}{\lambda}\eta+1\right) < \mathbb{E}_\varphi\left[\left(\frac{1}{\varphi}\right)^\eta\right]$. I assume this inequality is satisfied in order for the economy to exhibit capital outflows.

A short comment is in order. Whether this inequality holds or not depends on the consumption smoothing force embodied in the term $\frac{g_h-g}{\lambda}\eta+1$ on the one hand, and the precautionary motive reflected in $\mathbb{E}_\varphi\left[\left(\frac{1}{\varphi}\right)^\eta\right]$ on the other. First, for the standard case of a log-normal type distribution, $\log(\varphi) \sim N(-\frac{\sigma^2}{2}, \sigma^2)$, it is well known that $\mathbb{E}_\varphi\left[\left(\frac{1}{\varphi}\right)^\eta\right] = \exp\left(\frac{\sigma^2}{2}\eta\,[1+\eta]\right)$. The condition therefore reduces to $\sigma^2 > \frac{2}{\eta(1+\eta)}\log\left(1+\eta\frac{g_h-g}{\lambda}\right)$. For $\eta = 2$ any $\sigma^2$ above $0.296$ suffices, and for $\eta = 2.5$ any $\sigma^2$ above $0.234$.

However, the argument extends beyond the log-normal case. What is needed is the notion of a mean-preserving spread that can be applied in the context at hand. Note that Mas-Colell, Whinston, Green, et al. (1995) define a mean-preserving spread of a random variable $X$, based on the work of Diamond and Stiglitz (1974), in the following way: $X' = X + e$ s.t. $\mathbb{E}[e] = 0$. This does not work in the context at hand because I need to ensure that the type $\varphi$ is always greater zero. So if one wanted to define a mean preserving

[20] His results operate in a discrete time framework where shocks to permanent income are modeled as random walk with the error following a log normal distribution

spread, one would have to do something like $\varphi' = \varphi + e$ but this implies that $e \geq -\varphi$ which automatically induces statistical dependence among the random variables. When dealing with random variables that have to satisfy $\mathbb{E}[\varphi] = 1$, I propose the following version of a mean preserving spread $\varphi' = \varphi\epsilon$ with $\mathbb{E}[\epsilon\varphi] = \mathbb{E}[\epsilon]\mathbb{E}[\varphi] = 1$. Note that I have put zero distributional assumptions on neither $\varphi$ nor $\epsilon$ other than that they have to be unity in expectation. Now define $\varphi_k = \Pi_{j=1}^{k}\epsilon_j$, where the $\epsilon$'s are iid draws.

What I want to show is that for a sufficient amount of uncertainty about a household's type there exist a solution that sustains capital outflows, beyond log-normality. To see that this is the case, consider $\mathbb{E}[\varphi_k^{-\eta}]$. A higher $k$ here is a mean-preserving spread of the type distribution. Now note that

$$\begin{aligned}
\mathbb{P}\left(\varphi_K > y\right) &= \mathbb{P}\left(\Pi_{j=1}^{K}\epsilon_j < y\right) \\
&= \mathbb{P}\left(\sum_{j=1}^{K}\log(\epsilon_j) < \log(y)\right) \\
&= \mathbb{P}\left(\frac{\sum_{j=1}^{K}\log(\epsilon_j)}{K} < \frac{\log(y)}{K}\right) \\
&= \mathbb{P}\left(\hat{\mathbb{E}}_K \log(\epsilon_j) < \frac{\log(y)}{K}\right) \\
&= \mathbb{P}\left(\hat{\mathbb{E}}_K \log(\epsilon_j) < \frac{\log(y)}{K}\right)
\end{aligned}$$

where $y < 1$, and $\hat{\mathbb{E}}$ denotes the sample average. Note that because of Jensen's inequality it must be that $\mathbb{E}[\log(\epsilon)] < \log(\mathbb{E}[\epsilon]) = 0$ . Without loss of generality, assume $\mathbb{E}[\log(\epsilon)] = \frac{\log(y)}{M}$ for some $M > 0$. Now subtract the expectation of $\log(\epsilon)$

$$\begin{aligned}
\mathbb{P}\left(\varphi_K < y\right) &= \mathbb{P}\left(\hat{\mathbb{E}}_K \log(\epsilon_j) < \frac{\log(y)}{K}\right) \\
&= \mathbb{P}\left(\hat{\mathbb{E}}_K \log(\epsilon_j) - \mathbb{E}[\log(\epsilon)] < \frac{\log(y)}{K} - \mathbb{E}[\log(\epsilon)]\right) \\
&= \mathbb{P}\left(\hat{\mathbb{E}}_K \log(\epsilon_j) - \mathbb{E}[\log(\epsilon)] < -\log(y)\left(\frac{1}{M} - \frac{1}{K}\right)\right) \\
&> \mathbb{P}\left(\left|\hat{\mathbb{E}}_K \log(\epsilon_j) - \mathbb{E}[\log(\epsilon)]\right| < -\log(y)\left(\frac{1}{M} - \frac{1}{K}\right)\right)
\end{aligned}$$

Where the last inequality follows from the fact that $\{X : X < B\} = \{X : X \leq -B\} \cup \{X : -B < X < B\} \supset \{X : B < X < -B\} = \{X : |X| < B\}$.

$$\begin{aligned}
\mathbb{P}\left(\varphi_K < y\right) &> \mathbb{P}\left(\left|\hat{\mathbb{E}}_K \log(\epsilon_j) - \mathbb{E}[\log(\epsilon)]\right| < -\log(y)\left(\frac{1}{M} - \frac{1}{K}\right)\right) \\
&= 1 - \mathbb{P}\left(\left|\hat{\mathbb{E}}_K \log(\epsilon_j) - \mathbb{E}[\log(\epsilon)]\right| \geq -\log(y)\left(\frac{1}{M} - \frac{1}{K}\right)\right) \\
&\geq 1 - \frac{\mathbb{V}(\epsilon)}{K\left[-\log(y)\left(\frac{1}{M} - \frac{1}{K}\right)\right]^2}
\end{aligned}$$

where the last inequality follows from Chebyshev's inequality. Put differently, for large enough $K$ the probability of $\varphi$ being very small converges to one. It then follows that for large enough $K$, we have

$$\mathbb{E}\left[\left(\frac{1}{\varphi}\right)^{\eta}\right] > \mathbb{P}(\varphi < x)\left(\frac{1}{x}\right)^{\eta}$$

But I can make $\mathbb{P}(\varphi < x)\left(\frac{1}{x}\right)^{\eta}$ arbitrarily large as I am increasing $K$. Intuitively, the mean-preserving spread that I introduced shifts more and more mass on a very small $x$, but a small $x$ lets the expression explode. Given the results so far, it is easy to show that for any $y$ and for any number $L \in \{1, 2, ...\}$, I can find a $K$ such that $\mathbb{E}\left[\left(\frac{1}{\varphi_K}\right)^{\eta}\right] > L$. This concludes the treatment of the general case. Note that nowhere did I assume anything about the precise shape and support of the distribution. The purpose of this derivation was to show that the results do not rely on the particularities of the log normal distribution, or on the continuity of the type space. After obtaining this general result, I will focus on the case of log-normally distributed types since this the main exercise in the paper.

Next, I need to sign the derivative of the differential equations to draw the phase diagram, evaluated at the locus

$$\frac{d\frac{\dot{X}_s}{X_s}}{dZ}|_{X_{**}} < 0 \tag{23}$$

$$\frac{d\frac{\dot{Z}_s}{Z_s}}{dX}|_{Z_{**}} < 0. \tag{24}$$

Figure 12 displays the phase diagram. The dashed line represents the stable arm which is the unique trajectory of the system. Consumption is a control variable and jumps up when the household enters the high-growth regime so as to end up on the stable arm. From then on, the consumption-to-income ratio and the asset-to-income ratio increases till the steady state is reached. Consequently, consumption and assets grow at a rate higher than income, a standard result of models of precautionary savings.

**Law of motion out of agriculture in general case**

Next, I show that I can pin down the dynamics of the agricultural share even though I cannot solve for the transitional dynamics in the high growth regime in closed form. Once I establish this, we can conclude that an equilibrium exists, that is well behaved, in which the interaction of structural change, inequality, and growth generates capital outflows despite convergence growth.

From before, everything is captured in the asset-to-income and consumption-to-income ratio. Those dynamics are always the same, hence consumption and assets for households entering in the high growth regime at a later point in time are scaled up by a factor $\exp(gt)$ but otherwise identical.

To see this, note that the pseudo steady state analysis for an individual household always start at $t_m^i$, that is when the household leaves the country side. To see this, note that $\left\{\frac{c_t}{y_t}, \frac{b_t}{y_t}\right\}$ for households in the high-growth regime is only determined by the time spend in the high-growth regime $t - t_m$. This follows from the fact that the two (normalized) first order conditions do not depend on any variable that is a function of time $t$, the only thing one needs to keep track of here is $t - t_m = s$. It follows that the asset-to-income and consumption-to-income ratio are independent of calendar time, and only depend on the time spend in the high-growth regime. This property allows me to rewrite the consumption and asset profile of any agent in the high-growth regime as follows as follows:

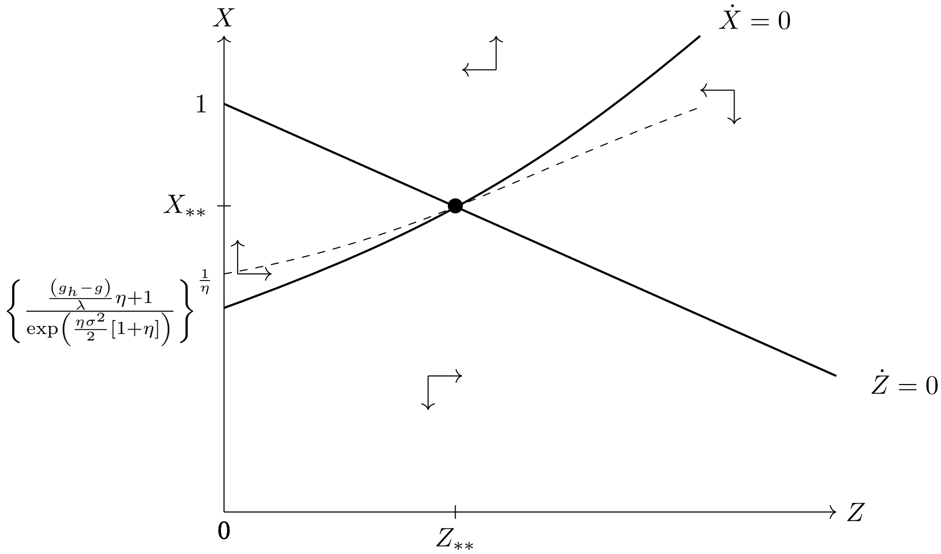


Figure 12. Pseudo steady state analysis of asset-to-income and consumption-to-income ratio in high-growth regime

$$
\begin{aligned}
c(t_m, t) &= c_0(t - t_m)\exp(gt_m) \\
a(t_m, t) &= b_0(t - t_m)\exp(gt_m) \\
y(t_m, t) &= \exp(g_h[t - t_m])\exp(gt_m) \\
&= y_0(t - t_m)\exp(gt_m)
\end{aligned}
$$

Proof:

$$X(s) = \frac{c(t, t_m)}{y(t, t_m)} = \frac{c(t + k, t_m + k)}{y(t + k, t_m + k)} \qquad \forall k \in [-t_m, \infty)$$

with a slight abuse of notation where $t_m + k$ represent the consumption profile of an agent that entered the city $k$ units of time later. Since the income process is exogenous, and every agent starts at the level $y(t_m) = A_t$, we can rewrite the equality

$$\frac{c(t, t_m)}{y(t - t_m, 0) A_{t_m}} = \frac{c(t - t_m, 0)}{y(t - t_m, 0)}$$

This implies that $c(t, t_m) = A_{t_m} c(t - t_m, 0)$ or short $c_0(t - t_m)\exp(gt_m)$. The case for assets is analogous. □

Of course, actual consumption and asset profiles need to be rescaled by income to obtain observed household asset holdings and consumption. The feature of the model that consumption and assets, after accounting for the time spend in the high-growth regime, can simply be scaled up by $A_{t_m}$ is key for tractability. As I show next, this allows me to derive the law of motion of workers out of the urban sector in closed form.

When considering the indifference condition that household on the country side consider before moving to the city, we can simplify this problem as follows. The value function reads

$$V_{t_m} = \max_{c_s} \mathbb{E}_{\varphi,T} \int_{t_m}^{\infty} \exp\left(-\rho[s-t_m]\right) \frac{c(\varphi,s,T)^{1-\eta}}{1-\eta} ds. \tag{25}$$

There is no simple solution for this expression in closed form. But, we can use the fact that all choices simply scale in income, a simplification that obtains from the CRRA utility function, in combination with the multiplicative type shock. Then we rescale consumption by $\exp(gt_m)$. This allows us to rewrite the expression as follows

$$\begin{aligned} V_{t_m} &= \exp\left(-g[\eta-1]t_m\right) \max_{c_s} \mathbb{E}_{\gamma,T} \int_{t_m}^{\infty} \exp\left(-\rho[s-t_m]\right) \frac{c_0(\varphi,s-t_m,T-t_m)^{1-\eta}}{1-\eta} ds & (26)\\ &= \exp\left(-g[\eta-1]t_m\right) V_0 & (27)\\ & & (28) \end{aligned}$$

This delivers the important result that

$$\dot{V_{t_m}} = -\left(g[\eta-1]\right) V_{t_m} \tag{29}$$

Two comments are in order. First, 29 only makes sense when $V_0$ is well defined. A sufficient condition for this to be the case is $\eta > 1 - \frac{\lambda+\rho}{g_h}$ and we need a finite expectation with respect to the type draw as well. Second, everything works out with log utility as well.[21] For finite utility in the low-growth regime, as well as in the industrialized world, we also need $\rho > [1-\eta]g$ which is assumed throughout the paper.

To finally pin down the law of motion out of the urban sector, we need to plug 29 into the indifference condition (3) that leaves a rural household indifferent between migrating and staying on the countryside. Conveniently, the time derivative is independent of $V_0$. Intuitively, migrants always face the same type of convergence process, the only difference is that the overall wage rate in the urban sector keeps growing.

$$\begin{aligned} \frac{\left(w_t^{\mathrm{R}}\right)^{1-\eta}}{1-\eta} &= \tau^{\eta-1}\left(\rho V_t - \dot{V}_t\right)\\ \frac{\left(w_t^{\mathrm{R}}\right)^{1-\eta}}{1-\eta} &= \tau^{\eta-1}\left(\rho V_t - g\left[1-\eta\right] V_t\right)\\ \frac{\left(w_t^{\mathrm{R}}\right)^{1-\eta}}{1-\eta} &= \tau^{\eta-1}\exp\left(g\left[1-\eta\right]t\right) V_0\left(\rho - g\left[1-\eta\right]\right)\\ w_t^{\mathrm{R}} &= \exp\left(gt\right)\frac{\left[\left(1-\eta\right)V_0\left(\rho+g\left[\eta-1\right]\right)\right]^{\frac{1}{1-\eta}}}{\tau}. \end{aligned}$$

Now, I only need to make use of the fact that the compensation on the country side is given by $w_t^{\mathrm{R}} =$

[21] Notes are available upon request.

$\left(L_t^{\mathrm{R}}\right)^{-[1-\chi]}$. Using this yields the share of households on the country side

$$L_t^{\mathrm{R}} = \{\tau\}^{\frac{1}{(1-\chi)}} \exp\left(-\frac{g}{1-\chi}t\right) \{V_0 [1-\eta] [\rho + g [\eta - 1]]\}^{\frac{1}{(1-\chi)(\eta-1)}}$$

And even though there is no closed form solution for $V_0$, to the extent that we observe the initial share of workers on the country side $L_0^{\mathrm{R}}$, we obtain a structural relationship based on observables

$$L_t^{\mathrm{R}} = L_0^{\mathrm{R}} \exp\left(-\frac{g}{1-\chi}t\right)$$

$\square$

I proceed by deriving the law of motion of households in the high and low growth regime, respectively. The convenient stochastic process delivers simple solutions. The labor resource constraint together with the Poisson process of drawing your type allows me to characterize the change in the different types $L_t^{\mathrm{U}}, M_{0,t}, M_{1,t}$ as laws of motion. First, note that from (31) we get

$$\frac{dL_t^{\mathrm{U}}}{dt} = -\frac{dL_t^{\mathrm{R}}}{dt}. \tag{30}$$

Moreover, the agents in the city that have drawn their type $M_{1,t}$ and the ones that haven't $M_{0,t}$ add up to $L_t^{\mathrm{U}}$,

$$L_t^{\mathrm{U}} = M_{0,t} + M_{1,t}, \tag{31}$$

and thus

$$\begin{aligned}\frac{dL_t^{\mathrm{U}}}{dt} &= \frac{dM_{0,t}}{dt} + \frac{dM_{1,t}}{dt} \\ &= \frac{dM_{0,t}}{dt} + \lambda M_{0,t},\end{aligned}$$

where the second line follows from the Poisson process, and the fact that there is a continuum of agents $M_{0,t}$. Rearranging yields the change in the fraction of agents that grow at the high rate $g_h$,

$$\frac{dL_t^{\mathrm{U}}}{dt} - \lambda M_{0,t} = \frac{dM_{0,t}}{dt}. \tag{32}$$

Whether this term is positive or negative depends on the relative strength of migration (inflow) and Poisson arrival process (outflow), and in the limit, $M_{1,\infty} = 1$. The change in the mass of agents that reside in the low-growth regime in the city is simply given by

$$dM_{1,t} = \lambda dt M_{0,t}. \tag{33}$$

using the Poisson arrival.[22]

The fraction of agents in the city at time zero is $1 - L_0^{\mathrm{R}}$. A share $M_{1,0}$ of them have already drawn their type; the remainder still have to draw it and start growing at the high rate. This would be agents in the set

[22] There is a law of large numbers operating in the background here.

$M_{0,0}$. Using (32) together with (11) yields

$$\dot{M}_{0,t} + \lambda M_{0,t} = \frac{g}{1-\chi} L_0^{\mathrm{R}} \exp\left(-\frac{g}{1-\chi}t\right). \tag{34}$$

Using $\exp(\lambda t)$ as integrating factor, this differential equation can be solved and yields

$$M_{0,t} = M_{0,0}\exp(-\lambda t) + \frac{g L_0^{\mathrm{R}}}{\lambda(1-\chi)-g}\left[\exp(-\frac{g}{1-\chi}t) - \exp(-\lambda t)\right]. \tag{35}$$

Similarly, the mass of households $M_{1,t}$ can be obtained

$$\begin{aligned} M_{1,t} &= \lambda \int_0^t M_{0,s} ds + M_{1,0} \\ &= M_{0,0}\left[1-\exp(-\lambda t)\right] + \left(\frac{g L_0^{\mathrm{R}}}{\lambda(1-\chi)-g}\right)\left\{\frac{\lambda(1-\chi)}{g}\left[1-\exp(-\frac{g}{1-\chi}t)\right] - \left[1-\exp(-\lambda t)\right]\right\} + M_{1,0} \end{aligned} \tag{36}$$

The changing shares of agents that don't know their type will be key to generate hump-shaped aggregate saving rates. Of course, in order to aggregate things up onto the macro level and get the aggregate savings in the economy right, we need to use appropriate weights that we attach to each household. Those weights will be based on the income of the household, which is why I study the dynamics of the income distribution next.

**Proof of Proposition 2:**

**[NOTE:** Write out the closed form here, since the main text now defers to it. With $\varphi \equiv 1$, $D \equiv g_h - r$ and $A \equiv [1 + \eta(g_h - g)/\lambda]^{1/\eta}$, the loci $X = A(1+\kappa Z)$ and $X = 1 - DZ$ give $Z_{**} = (1-A)/(D + A\kappa)$ and $X_{**} = A(D+\kappa)/(D+A\kappa)$, so $X_{**} > 1 \iff A > 1 \iff g_h > g$. At the baseline calibration $X_{**} = 1.17$ and $Z_{**} = -8.37$, the values quoted in the footnote in Section 3.4. Also state that the steady state is a saddle (eigenvalues $+0.222$, $-0.072$) and that $X$ and $Z$ are monotone along the path, which is what the proposition needs and what the current text only asserts.**]**

*Proof.* As argued before, the phase diagram analysis shows that the steady state as well as the transition path display $X_s > 1$. Clearly, if the consumption-to-income ratio is greater unity at all times in the high-growth regime there must be capital inflows to finance the gap between output and consumption. □

**Proposition 4** (Biased expectations). *Without human capital risk, there are no capital outflows, even if expectations about convergence are downward biased, $\tilde{\lambda} > \lambda$, where $\tilde{\lambda}$ is the household's belief about the arrival rate of the low-growth regime.*

**Proof of Proposition 4:**

*Proof.* Note that the proof for proposition 4 follows simply from the fact that the consumption-growth ranking (now a corollary of the sign of the Euler bracket, Section 3.4) and Proposition 2 hold for any positive and finite $\lambda$.

□

**Migration Decision:**

Given that agents are on the conjectured equilibrium, I can solve the arbitrage condition that pins down the flow agents into the city. Recall

$$log(w_t^{\mathrm{R}}) \stackrel{!}{=} \rho V_t - \dot{V}_t.$$

Given the conjectured equilibrium, I obtain a closed form solution for the value function as follows.

$$\begin{aligned}
V_{t,0} &= \int_t^{\infty} \exp(-(\lambda+\rho)[s-t]) \left\{log(y_s) + \lambda \mathbb{E}_{\varphi} V_{s,1}(\varphi, b_s)\right\} ds \\
&= \int_t^{\infty} \exp(-(\lambda+\rho)[s-t]) \left\{[log(y_t) + g_h[s-t]] + \lambda \left[\frac{log(y_s)}{\rho} + \mathbb{E}_{\varphi}\left[\frac{log(\varphi)}{\rho}\right] + \frac{g}{\rho^2}\right]\right\} ds \\
&= \int_t^{\infty} \exp(-(\lambda+\rho)[s-t]) \left\{[log(y_t) + g_h[s-t]] + \lambda \left[\frac{log(y_s)}{\rho} + \mathbb{E}_{\varphi}\left[\frac{log(\varphi)}{\rho}\right] + \frac{g}{\rho^2}\right]\right\} ds \\
&= \int_t^{\infty} \exp(-(\lambda+\rho)[s-t]) \left\{[log(y_t) + g_h[s-t]] + \lambda \left[\frac{log(y_s)}{\rho} - \frac{\sigma^2}{2\rho} + \frac{g}{\rho^2}\right]\right\} ds \\
&= \int_t^{\infty} \exp(-(\lambda+\rho)[s-t]) \left\{[log(y_t) + g_h[s-t]]\,(1+\frac{\lambda}{\rho}) + \lambda \left[\frac{g}{\rho^2} - \frac{\sigma^2}{2\rho}\right]\right\} ds \\
&= \left[\frac{log(y_t)}{\lambda+\rho} + \frac{g_h}{(\lambda+\rho)^2}\right](1+\frac{\lambda}{\rho}) + \frac{\lambda}{\lambda+\rho}\left[\frac{g}{\rho^2} - \frac{\sigma^2}{2\rho}\right] \\
&= \left[\frac{log(y_t)}{\lambda+\rho}\right](1+\frac{\lambda}{\rho}) + \frac{1}{\lambda+\rho}\frac{1}{\rho^2}\left[\lambda g + \rho g_h\right] - \frac{\lambda}{\lambda+\rho}\frac{\sigma^2}{2\rho} \\
&= \frac{log(y_t)}{\rho} + \frac{1}{\lambda+\rho}\frac{1}{\rho^2}\left[\lambda g + \rho g_h\right] - \frac{\lambda}{\lambda+\rho}\frac{\sigma^2}{2\rho}.
\end{aligned}$$

Now I can differentiate this expression with respect to $t$, and plug it back into the arbitrage condition that keeps agents on the country side indifferent between staying and moving, hence

$$\begin{aligned}
log(w_t^{\mathrm{R}}) &= \rho\left(\frac{log(y_t)}{\rho} + \frac{1}{\lambda+\rho}\frac{1}{\rho^2}\left[\lambda g + \rho g_h\right] - \frac{\lambda}{\lambda+\rho}\frac{\sigma^2}{2\rho}\right) - \dot{V}_t \\
&= log(y_t) + \frac{1}{\lambda+\rho}\frac{1}{\rho}\left[\lambda g + \rho g_h\right] - \frac{\lambda}{\lambda+\rho}\frac{\sigma^2}{2} - \dot{V}_t \\
&= log(y_t) + \frac{1}{\lambda+\rho}\frac{1}{\rho}\left[\lambda g + \rho g_h\right] - \frac{\lambda}{\lambda+\rho}\frac{\sigma^2}{2} - \frac{g}{\rho} \\
&= log(A_t) + \frac{1}{\lambda+\rho}(g_h - g) - \frac{\lambda}{\lambda+\rho}\frac{\sigma^2}{2}.
\end{aligned}$$

### 6.1.1 agricultural productivity gap/urban rural wage gap

There is a link between the model and the literature on the urban rural wage gap (Harris and Todaro, 1970; Young, 2013; Lagakos and Waugh, 2013; Hicks et al., 2017) and the agricultural productivity gap (Caselli, 2005; Restuccia, Yang, and Zhu, 2008; Gollin, Lagakos, and Waugh, 2013). In the model, ex post inequality in the city can drive a wedge between urban and rural wages, and reduce the share of people working in urban areas relative to a world with complete markets. To see this, I solve a version of the model in log utility where the precautionary savings and the consumption smoothing motive exactly offset each other. Then, I get a closed form solution of the value function, and can pin down the rural wage as a function of the state of the technology in the city, as well as convergence growth and inequality, $\sigma^2$. This relationship is captured in equation (37)

$$\log(w_t^{\mathrm{R}}) = \log(A_t) + \frac{1}{\lambda+\rho}\left(g_h - g\right) - \frac{\lambda}{\lambda+\rho}\frac{\sigma^2}{2} - \log\left(\tau\right). \tag{37}$$

In my model there is no selection on skill (ex ante everyone is the same!) but an econometrician that would compare labor productivity approximated by average log wages in a cross section of workers may conclude that there is an agricultural productivity gap, while the actual reason is a compensating risk differential. Wage growth in the rural region will be identical to wage growth in the city, and since there is potential for convergence growth in the city in contrast to the country side, rural workers are compensated for that by the term $\frac{1}{\lambda+\rho}\left(g_h - g\right)$. Note that without the wedge, assuming that convergence growth and precautionary savings cancel, it can be shown that the urban wage at time $t_m$ would be smaller than the rural wage at $t_m$. This happens because rural households need to be compensated for the lack of high-growth opportunity in equilibrium. Given log utility, this force dominates the risk adjustment, that pushed down the rural wage. This is why we need $\tau > 1$, i.e. there needs to be an additional wedge to migration.

## 6.2 Derivation of the income distribution

Log income net of the common trend depends on calendar time only through the time a household has already spent in the high-growth phase. Write that duration as

$$x_i(t) = \begin{cases} t - t_{m,i}, & i \in M_{0,t}, \\ T_i - t_{m,i}, & i \in M_{1,t}, \end{cases}$$

and let $\Delta \equiv g_h - g$ and $z_{i,t} \equiv \log y_{i,t} - \log A_t^{\mathrm{U}}$, so that

$$z_{i,t} = \Delta x_i(t) \quad \text{for } i \in M_{0,t}, \qquad z_{i,t} = \Delta x_i(t) + \log\varphi_i \quad \text{for } i \in M_{1,t}.$$

The pair $(t_{m,i}, T_i)$ enters only through its difference, so the entire cross-section at date $t$ is described by the distribution of a single scalar within each of the two groups. This is what makes the transition path tractable. Throughout write $\nu \equiv \frac{g}{1-\chi}$ for the rate at which the rural sector empties, so that $L_t^{\mathrm{R}} = L_0^{\mathrm{R}} e^{-\nu t}$ and the flow of migrants at date $s$ is $\nu L_0^{\mathrm{R}} e^{-\nu s}$.

**Lemma 1** (Duration densities)**.** *At date $t$ the mass of high-growth households with duration in $[x, x+dx]$ is $h_{0,t}(x)dx$, where*

$$h_{0,t}(x) = \nu L_t^{\mathrm{R}}\, e^{(\nu-\lambda)x}, \qquad x \in (0,t), \tag{38}$$

*together with an atom of size $M_{0,0}e^{-\lambda t}$ at $x = t$. The mass of typed households with completed duration $x$ is*

$$h_{1,t}(x) = \lambda\left(1 - M_{1,0}\right)e^{-\lambda x} - \lambda L_t^{\mathrm{R}}\, e^{(\nu-\lambda)x}, \qquad x \in (0,t), \tag{39}$$

*together with an atom of size $M_{1,0}$ at $x = 0$.*

*Proof.* A household in the high-growth phase at $t$ with duration $x$ migrated at $s = t - x$ and has not drawn its type since. Its mass is the migration flow at $t - x$ times the survival probability $e^{-\lambda x}$,

$$\nu L_0^{\mathrm{R}} e^{-\nu(t-x)} e^{-\lambda x} = \nu L_t^{\mathrm{R}} e^{(\nu-\lambda)x}.$$

Households already urban at date zero are treated as having entered at date zero. The mass $M_{0,0}e^{-\lambda t}$ of them still in the high-growth phase at $t$ all have duration exactly $t$, which is the atom.

A typed household with completed duration $x$ entered at some date $s \le t-x$ and drew its type at $s+x$. Integrating the exit hazard over entry dates,

$$h_{1,t}(x) = \lambda e^{-\lambda x}\left[M_{0,0} + \int_0^{t-x} \nu L_0^{\mathrm{R}} e^{-\nu s} ds\right] = \lambda e^{-\lambda x}\left[M_{0,0} + L_0^{\mathrm{R}}\left(1 - e^{-\nu(t-x)}\right)\right],$$

and $M_{0,0} + L_0^{\mathrm{R}} = 1 - M_{1,0}$ delivers (39). The atom at $x = 0$ is the initial mass $M_{1,0}$, which never passes through the high-growth phase. A law of large numbers operates within each cohort, as is standard in this class of models (Luttmer, 2007; Arkolakis, 2010). □

Integrating the two densities returns the masses derived above, which is a useful check on the algebra. For the high-growth group,

$$\int_0^t h_{0,t}(x)dx + M_{0,0}e^{-\lambda t} = M_{0,0}e^{-\lambda t} + \frac{gL_0^{\mathrm{R}}}{\lambda(1-\chi)-g}\left[e^{-\frac{g}{1-\chi}t} - e^{-\lambda t}\right] = M_{0,t},$$

which is (35), and the typed group integrates to $M_{1,t}$ in (36) in the same way.

**Income densities.** Setting the type draw aside for a moment, income relative to entry is $y = e^{\Delta x}$, so a change of variable with $dx/dy = 1/(\Delta y)$ delivers the level densities in one step,

$$f_{0,t}(y) = \frac{h_{0,t}\left(\log y/\Delta\right)}{\Delta y} = \frac{\nu L_t^{\mathrm{R}}}{\Delta} y^{-\frac{\lambda-\nu}{\Delta}-1}, \qquad y \in \left(1, e^{\Delta t}\right), \tag{40}$$

$$f_{1,t}(y) = \frac{h_{1,t}\left(\log y/\Delta\right)}{\Delta y} = \frac{\lambda}{\Delta} y^{-\frac{\lambda}{\Delta}-1}\left[1 - M_{1,0} - L_t^{\mathrm{R}} y^{\frac{\nu}{\Delta}}\right], \tag{41}$$

both unnormalized, so that dividing by $M_{0,t}$ and $M_{1,t}$ gives the densities conditional on group membership. Both are power laws, the familiar consequence of exponential growth sustained for an exponentially distributed length of time (Luttmer, 2011; Gabaix et al., 2016). Since $L_t^{\mathrm{R}} \to 0$, the tail index of the limiting distribution is $\lambda/\Delta$, the value used in Section 4.

**The distribution of log income.** The same two densities deliver the full distribution of log income at every date, which is the object I take to the data. For the high-growth group $z = \Delta x$ has support $[0, \Delta t]$, so that

$$F_{0,t}(z) = \frac{\nu L_t^{\mathrm{R}}}{\nu - \lambda}\left[e^{(\nu-\lambda)\min\{z/\Delta,\, t\}} - 1\right] + M_{0,0}e^{-\lambda t}\,\mathbf{1}\left\{z \ge \Delta t\right\}, \qquad z \ge 0, \tag{42}$$

and $F_{0,t}(z) = 0$ for $z < 0$. For the typed group $z = \Delta x + \log\varphi$ with $\log\varphi \sim N(-\sigma^2/2, \sigma^2)$ drawn independently of $x$, so that with $\Phi$ the standard normal CDF and

$$\beta \equiv \frac{z + \sigma^2/2}{\sigma}, \qquad \alpha \equiv \beta - \frac{\Delta t}{\sigma},$$

we have

$$F_{1,t}(z) = M_{1,0}\Phi\left(\beta\right) + \lambda\left(1 - M_{1,0}\right)\mathcal{I}\left(\lambda; z\right) - \lambda L_t^{\mathrm{R}}\mathcal{I}\left(\lambda - \nu; z\right), \tag{43}$$

where $\mathcal{I}(a; z) \equiv \int_0^t e^{-ax}\Phi\left(\beta - \frac{\Delta x}{\sigma}\right)dx$. Integrating by parts and completing the square in the resulting

Gaussian integral gives

$$\mathcal{I}(a;z) = \frac{1}{a}\left[\Phi(\beta) - e^{-at}\Phi(\alpha)\right] - \frac{1}{a}\exp\left(-\frac{a}{\Delta}\left(z+\frac{\sigma^2}{2}\right)+\frac{a^2\sigma^2}{2\Delta^2}\right)\left[\Phi\left(\beta-\frac{a\sigma}{\Delta}\right)-\Phi\left(\alpha-\frac{a\sigma}{\Delta}\right)\right]. \quad (44)$$

This is an exponentially modified Gaussian: a normal type draw convolved with an exponentially distributed length of exposure to high growth. Collecting terms, the distribution of log income among urban households at date $t$ is

$$\Pr(z_{i,t} \le z) = \frac{F_{0,t}(z) + F_{1,t}(z)}{M_{0,t} + M_{1,t}}. \quad (45)$$

The distribution is available in closed form at every date, and it moves over time both because the two group masses move and because durations accumulate within each group. Moments follow from Lemma 1 directly rather than from (45). In the high-growth group the mean and variance of log income are $\Delta\,\mathbb{E}[x]$ and $\Delta^2\,\mathrm{Var}(x)$ taken under $h_{0,t}/M_{0,t}$, an exponential density with rate $\lambda-\nu$ truncated at $t$ plus the atom at $x=t$; in the typed group the type draw adds $-\sigma^2/2$ to the mean and $\sigma^2$ to the variance, since it is independent of duration.

## 6.3 A version with even catch-up growth

To see how heterogeneous and risky income growth is central, consider a model economy where convergence growth is deterministic and occurs up until time $T$, when the households draw their type $\varphi$ as before. I hold the aggregate level of convergence fixed but distribute the growth it takes to get there evenly. Foreshadowing a calibration exercise in the next section, I require that aggregate urban income per capita grows by the same factor as in the baseline economy. Netting out the wage gap, that factor is $0.825 \times 3.5 + 0.175 = 3.0625$, since the share $M_{1,0} = .175$ of the initial urban population does not pass through the high-growth regime; the per-migrant multiplier of $3.5$ is the larger number and is not the right target here. With $g_h = .07$ and $g = .02$, an interest rate of 5% and a discount factor $\rho$ of .01, this means that $T = \frac{\log(3.0625)}{g_h - g} \approx 22.4$. Noting that the household optimality condition leads to an equalization of (expected) marginal utilities, and in particular at time $T$ with $\Delta \to 0$,

$$c_{T-\Delta}^{-\eta} = \mathbb{E}_\varphi \left[ (\varphi y_T + (\rho + [\eta - 1]g) b_T)^{-\eta} \right],$$

I can ask what level of inequality, here in the form of the variance of the type draw $\sigma^2$, is needed to generate capital outflows along the transition path. Equivalently, this asks what level of variance is needed to push the intercept of the consumption profile below $y_0$. This follows since the slope of the consumption profile is only pinned down by preference parameters and the interest rate in this version of the model[23]. Consequently, the optimality condition simplifies to

$$\exp\left(\eta \left[g_h - g\right] T\right) = \mathbb{E}_\varphi \left[ \left( \varphi^i + \exp\left(\left[r - g_h\right] T\right) \left[ \frac{r - g}{g_h - r} \left[\exp\left(\left[g_h - r\right] T\right) - 1\right] - \left[1 - \exp\left(\left[g - r\right] T\right)\right] \right] \right)^{-\eta} \right]$$

where I used the fact that income and consumption grow at rate $g_h$ and $g$, as well as the budget constraint and the requirement that $y_0 = c_0$. Assuming that the type draw is log normal, the variance of the log of the type that is needed to solve this equation is a staggering $4.75$, and completely out of range of any empirically sensible estimate of income dispersion.[24] This highlights the importance of the stochastic nature of the catch-up growth on the household level, not just for tractability but also to quantitatively account for household asset accumulation despite strong convergence growth.

There are two additional remarks worth pointing out. First, note the tension between inter-temporal and intra-temporal smoothing. In a world where every household converges to some average level, a large coefficient of relative risk aversion will induce a strong consumption smoothing motive. Since lifetime utility becomes increasingly defined by the lowest level of consumption as $\eta$ increases, households want to smooth consumption and borrow against their future lifetime income. This happens in the case with deterministic convergence. In a world with risky growth, however, this logic is turned upside down. If households are very risk averse, they effectively attach more weight to the worst convergence growth path inducing them to built up savings.

[23] This simplified model is inconsistent with the strong comovement of consumption and output in the data as mentioned before. Output growth and consumption growth track each other.

[24] In this simplified version of the model, the type draw is the only source of uncertainty. Inequality measured in terms of the log of income therefore would exceed standard measures of labor income inequality of .6 and household income inequality of around 1 (Krueger, Mitman, and Perri, 2016).

### 6.4 A model version with capital

Here I introduce a simple version of the model with capital that leaves all qualitative conclusions unchanged. First, I reinterpret what used to be the labor supply in the rural economy $l_r^i$ of household $i$ as a composite intermediate input that uses both raw rural labor and capital as inputs in a Cobb-Douglas fashion with capital elasticity $\beta$. Hence, $x_r^i = k_i^\beta l_i^{1-\beta}$. Rural total output is now given by $A^{\mathrm{R}} X^\chi$. Suppose that rural households save and invest a fraction $s$ of their income to buy more capital, as in Solow (1956). Suppose that as before, the composite factor $x$ flows out of the rural economy at a rate such that there is constant income growth for rural households at rate $g$.

Now consider the capital accumulation of household $i$,

$$\begin{aligned} k_i &= s y_i \\ &= s A^{\mathrm{R}} X^{\chi - 1} \end{aligned}$$

Now focus on the balanced growth path where capital is accumulated at rate $g$. The steady state level is of course endogenous and depends on the saving rate and productivity etc. As before, normalize $l_i$ to unity. Suppose, for the sake of the argument, that the steady state capital–effective urban labor ratio is such that $\beta \left( \frac{k_t^i}{A_t^{\mathrm{U}}} \right)^{\beta - 1} = r$.

This choice is motivated by the desire to ensure that every household that leaves the rural sector brings a sufficient amount of capital with them so that the influx of workers into the urban sector does not raise the marginal product of capital. Note that if entering workers were to enter without any capital, we would have two offsetting forces on the direction of capital flows. On the one hand, entering households will increase the marginal product of capital – a simple labor supply shock that should lead to capital inflows. On the other hand, precautionary savings are accumulated, potentially inducing outflows. It is ex ante unclear which force dominates. In this modified version, however, every worker enters the city with a sufficient amount of capital to ensure that the marginal product of capital is left unchanged. As before, miracle growth increases the effective units of labor of each household. But as long as $k_{t_m}^i < b_{t_m}^{**}$, we know that the household will accumulate assets at a rate that is higher than their labor income growth. Simply put, the household problem has not changed, except now the household does not start with an asset-to-income ratio that is zero but with one that is large enough to match the labor supply they are contributing to the urban economy. Whether, in fact, the desired bufferstock asset-to-income ratio is larger than the amount of capital already owned by the household depends on the parameters of the model, especially the rural saving rate $s$ as well as the risk in the urban economy. If there is no risk in the form of the type draw, we know that this inequality is never going to hold. If there is a sufficient amount of risk, this may well be the case.

Lastly, the reader may worry that the type draw itself creates some complications by raising the marginal product of capital. Since production is constant returns to scale, it is easiest to consider one household first, and aggregate up in a last step

$$y_t^i = k_t^\beta \left( A_t \varphi h_t \right)^{1-\beta} \tag{46}$$

The allocation of capital to each household unit is simply given by the first order condition with respect to $k$

$$A_t \varphi h_t \left( \frac{\beta}{r} \right)^{\frac{1}{1-\beta}} = k_t$$

Note that whenever a fraction of households draws their type, there is no jump in the aggregate demand for capital since the type draw is centered around one. Capital can be reshuffled within each cohort of agents drawing their type while leaving the overall demand for capital in the economy unchanged. This concludes the generalization of the model, showing that it is in principal able to accommodate the inclusion of capital as a factor of production.

# 7 Empirical Appendix

## 7.1 CFPS

The CFPS is a household panel dataset that comprises detailed information on family structure, income, expenditure, assets, and demographics. The survey was launched in 2010 by the Institute of Social Science Survey (ISSS) of Peking University, China.[25] The dataset is similar to the PSID, but many survey questions are designed to capture relevant variables for Chinese families. The CFPS data for 2010 contains roughly 15,000 households, in 25 provinces excluding Hong Kong, Macao, Tibet, Qinghai, Inner Mongolia, Ningxia, Hainan. An eligible household refers to an independent economic unit with at least one Chinese national. I use the CFPS to study differences in household savings and asset-accumulation behavior across urban and rural areas.

**Concept of a Household/Family**

Every household in the CPFS has at least one Chinese national, and the family is defined as interdependent economic unit. Household members are defined as financially dependent immediate relatives, or non-immediate relatives who lived with the household for more than three consecutive months and are financially related to the sampled household. That includes families with household members who migrated for work to another city. It does not include family members that got married and started their own family. In the panel dimension, households are "split up" to keep track of those changes and follow the different household units over time. Ideally, the household unit therefore also incorporates migrant workers. A feature of the sample that has been exploited by Xu and Xie (2015).

[25] After applying for access online, the data is in principle accessible to any researcher. For more information, see https://opendata.pku.edu.cn/dataverse/CFPS?language=en.

### Descriptive Statistics CFPS

Table 5 and 6 display mean and standard deviations of the raw sample, i.e. before I restrict on age, labor market attachment, and household splits, and for the selected sample, respectively.

Table 5. Descriptive Statistic CFPS

| | Rural mean | sd | Urban mean | sd |
|---|---|---|---|---|
| Broad labor income | 38915.2 | 85560.3 | 64739.3 | 81798.0 |
| Years of schooling | 6.1 | 4.3 | 9.0 | 4.7 |
| Age | 50.8 | 13.8 | 49.3 | 15.1 |
| Male share | 0.7 | 0.4 | 0.7 | 0.5 |
| Household size | 4.0 | 1.9 | 3.4 | 1.7 |
| Total net assets | 236532.5 | 835413.0 | 713546.6 | 1582119.2 |
| Safe assets | 21651.6 | 72873.4 | 74205.3 | 249065.8 |
| Observations | 25556 | | 24179 | |

*Note*: Mean and standard deviation for the raw sample, i.e. before selection.

Table 6. Descriptive Statistic CFPS – Selected Sample

| | Rural mean | sd | Urban mean | sd |
|---|---|---|---|---|
| Broad labor income | 38660.9 | 42847.5 | 65632.3 | 86969.4 |
| Years of schooling | 6.2 | 4.1 | 9.4 | 4.4 |
| Age | 46.3 | 9.1 | 45.2 | 9.1 |
| Male share | 0.7 | 0.4 | 0.7 | 0.5 |
| Household size | 4.3 | 1.7 | 3.7 | 1.5 |
| Total net assets | 242997.9 | 698359.2 | 711700.6 | 1361748.5 |
| Safe assets | 21988.5 | 68107.3 | 79621.3 | 275307.8 |
| Observations | 12132 | | 10468 | |

*Note*: Mean and standard deviation for the selected sample.

### Additional Quantile Regression: total assets and sample weights

Table 7. Rural-Urban Median Differences for Safe Assets, Sampling Weights

| | (1) safe_asset_income_ratio | (2) safe_asset_income_ratio | (3) safe_asset_income_ratio |
|---|---|---|---|
| urban dummy | $0.208^{***}$ | $0.191^{***}$ | $0.128^{***}$ |
| | (0.0296) | (0.0285) | (0.0288) |
| _cons | $0.330^{***}$ | $0.806^{**}$ | 0.357 |
| | (0.0155) | (0.394) | (0.247) |
| age | No | Yes | Yes |
| sex and hh size | No | No | Yes |
| education | No | No | Yes |
| Observations | 5384 | 5384 | 5384 |

*Note*: The dependent variable is the household financial asset-to-income-ratio. This contains bank deposits, stocks, derivatives, bonds, cash, and other financial assets. Robust Standard errors in parentheses. *, **,*** denote statistical significance at 1, 5, and 10 percent level. I apply cross-sectional weights for 2012 for this regression.

Table 8. Rural-Urban Median Differences for Total Assets

| | (1) tot_asset_income_ratio | (2) tot_asset_income_ratio | (3) tot_asset_income_ratio |
|---|---|---|---|
| urban dummy | 2.780*** | 2.849*** | 2.494*** |
| | (0.204) | (0.206) | (0.232) |
| _cons | 5.843*** | 6.874*** | 7.635*** |
| | (0.102) | (2.498) | (2.370) |
| age | No | Yes | Yes |
| sex and hh size | No | No | Yes |
| education | No | No | Yes |
| Observations | 5949 | 5949 | 5937 |

*Note*: The dependent variable is the household financial asset-to-income-ratio. This contains bank deposits, stocks, derivatives, bonds, cash, and other financial assets. Robust Standard errors in parentheses. *, **,*** denote statistical significance at 1, 5, and 10 percent level.

Table 9. Rural-Urban Median Differences for Total Assets, Sampling Weights

| | (1) tot_asset_income_ratio | (2) tot_asset_income_ratio | (3) tot_asset_income_ratio |
|---|---|---|---|
| urban dummy | 2.151*** | 2.154*** | 1.780*** |
| | (0.267) | (0.250) | (0.264) |
| _cons | 6.085*** | 14.65*** | 14.41*** |
| | (0.148) | (3.566) | (3.585) |
| age | No | Yes | Yes |
| sex and hh size | No | No | Yes |
| education | No | No | Yes |
| Observations | 5384 | 5384 | 5384 |

*Note*: The dependent variable is the household financial asset-to-income-ratio. This contains bank deposits, stocks, derivatives, bonds, cash, and other financial assets. Robust Standard errors in parentheses. *, **,*** denote statistical significance at 1, 5, and 10 percent level. I apply cross-sectional weights for 2012 for this regression.